PHYSICS

SECTION 1 (Maximum Marks: 12)

- This section contains THREE (03) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 ONLY if (all) the correct option(s) is(are) chosen;
  Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;
  Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;
  Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : -2 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks; choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks; choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark; choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option (i.e. the question is unanswered) will get 0 marks; and choosing any other combination of options will get -2 marks.

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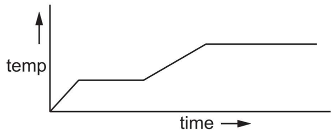
1. 

Heat is supplied to a certain homogenous sample of matter, at a uniform rate. Its temperature is plotted against time, as shown. Which of the following statement/statements are true?
(a) Its specific heat capacity is greater in the solid state than in the liquid state.
(b) Its specific heat capacity is greater in the liquid state than in the solid state.
(c) Its latent heat of vaporization is greater than its latent heat of fusion.
(d) Its latent heat of vaporization is smaller than its latent heat of fusion.

Sol. The horizontal parts of the curve, where the system absorbs heat at constant temperature, must depict changes of state. Here the latent heats are proportional to the lengths of the horizontal parts. In the sloping parts, specific heat capacity is inversely proportional to the slopes.

2. A plane progressive wave of frequency 25 Hz, amplitude 2.5×10^{-5} m and initial phase zero moves along the negative x-direction with a velocity of 300 m/s. A and B are two points 6 m apart on the line of propagation of the wave. At any instant the phase difference between A and B is ϕ. The maximum difference in the displacements at A and B is Δ.

(A) $\phi = \pi$ (B) $\phi = \frac{\pi}{2}$ (C) $\Delta = 2.5 \times 10^{-5}$ **(D)** $\Delta = 5 \times 10^{-5}$ m

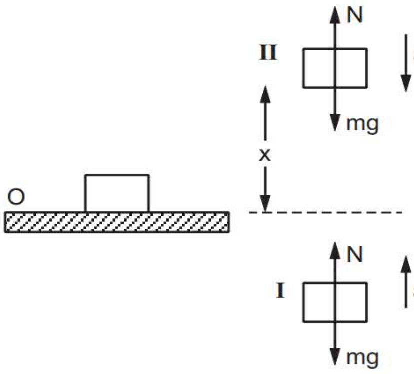
Sol. $\lambda = \frac{300 m/s}{25 Hz} = 12m.$

Separation between A and B = 6 m = $\frac{\lambda}{2}$.

3. A coin is placed on a horizontal platform, which undergoes vertical simple harmonic motion of angular frequency ω. The amplitude of oscillation is gradually increased. The coin will leave contact with the platform for the first time
(a) at the highest position of the platform (b) at the mean position of the platform

(c) for an amplitude of g/ω^2 (d) for an amplitude of $\frac{\sqrt{g}}{\omega}$

Sol.



Let O be the mean position and a be the acceleration at a displacement x from O.

At position I, N - mg = ma. $\therefore N \neq 0$

At position II, mg - N = ma.

For N = 0 (loss of contact), $g = a = \omega^2 x.$

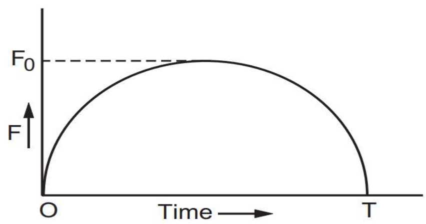
Loss of contact will occur for amplitude $x_{max} = \frac{g}{\omega^2}$ at the highest point of the motion.

SECTION 2 (Maximum Marks: 12)

- This section contains FOUR (04) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : -1 In all other cases.

4. A particle of mass m, initially at rest, is acted upon by a variable force F for a brief interval of time T. It begins to move with a velocity u after the force stops acting. F is shown in the graph as a function of time. Value of u will be

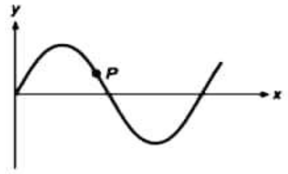


(A) $u = \frac{\pi F_0^2}{2m}$ (B) $u = \frac{\pi T^2}{8m}$ **(C)** $u = \frac{\pi F_0 T}{4m}$ (D) $u = \frac{\pi F_0 T}{2m}$

Sol. Area of F-T curve gives $\Delta\vec{p}$

$$\Rightarrow mu = \frac{1}{2}\left(\pi \times F_0 \times \frac{T}{2}\right) \Rightarrow u = \frac{\pi F_0 T}{4m}$$

5. A transverse sinusoidal wave moves along a string in the positive x-direction at a speed of 10 cm/s. The wavelength of the wave is 0.5 m and its amplitude is 10 cm. At a particular time t, the snapshot of the wave is shown in figure. The velocity of point P when its displacement is 5 cm is



(A) $\frac{\sqrt{3}\pi}{50}\hat{j}$ m/s (B) $-\frac{\sqrt{3}\pi}{50}\hat{j}$ m/s (C) $\frac{\sqrt{3}\pi}{50}\hat{i}$ m/s (D) $-\frac{\sqrt{3}\pi}{50}\hat{i}$ m/s

Sol. Particle velocity v_P = -v(slope of y-x graph)
Here, v = +ve, as the wave is travelling in positive x-direction. Slope at Pis negative.
Velocity of particle is in positive y(or $\hat{j}$) direction.
Correct option is (a).

6. The work done on a particle of mass m by a force $K\left[\frac{x}{(x^2+y^2)^{3/2}}\hat{i}+\frac{y}{(x^2+y^2)^{3/2}}\hat{j}\right]$ (K being a constant of appropriate dimensions, when the particle is taken from the point (a, 0) to the point (0, a) along a circular path of radius a about the origin in the x-y plane is

(a) $\frac{2K\pi}{a}$ (B) $\frac{K\pi}{a}$ (C) $\frac{K\pi}{2a}$ **(D)** 0

Sol. $dw = \vec{F}\cdot d\vec{r} = \vec{F}\cdot(dx\hat{i}+dy\hat{j}) = K\int \frac{xdx}{(x^2+y^2)^{3/2}} + \frac{ydy}{(x^2+y^2)^{3/2}}$

$x^2 + y^2 = a^2$

$w = \frac{K}{a^3}\int_a^0 xdx + \int_0^a ydy = \frac{K}{a^3}\left(\frac{-a^2}{2}+\frac{a^2}{2}\right) = \mathbf{0.}$

7. Two non-reactive monoatomic ideal gases have their atomic masses in the ratio 2 : 3. The ratio of their partial pressures, when enclosed in a vessel kept at a constant temperature, is 4 : 3. The ratio of their densities is
(A) 1 : 4 (B) 1 : 2 (C) 6 : 9 (D) 8 : 9

Sol. $PV = nRT = \frac{m}{M}RT$

$\Rightarrow PM = \rho RT$

$\frac{\rho_1}{\rho_2} = \frac{P_1M_1}{P_2M_2} = \left(\frac{P_1}{P_2}\right)\times\left(\frac{M_1}{M_2}\right) = \frac{4}{3}\times\frac{2}{3} = \frac{8}{9}$

Here ρ_1 and ρ_2 are the densities of gases in the vessel containing the mixture.

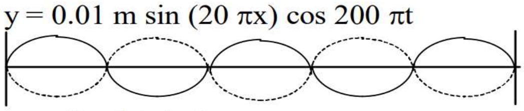
SECTION 3 (Maximum Marks: 24)

- This section contains SIX (06) questions.
- The answer to each question is a NON-NEGATIVE INTEGER.
- For each question, enter the correct integer corresponding to the answer on OMR sheet.

- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 If ONLY the correct integer is entered;
  Zero Marks : 0 In all other cases.

8. A horizontal stretched string fixed at two ends, is vibrating in its fifth harmonic according to the equation $y(x,t) = 0.01m\sin[(62.8m^{-1})x]\cos[(628s^{-1})t]$. If ℓ be the length of the string, n be the number of nodes of the wave produced and p be the maximum displacement of the midpoint of the string from the equillibrium position. Calculate n + 100 ℓ + 1000p = ______(Take $\pi = 3.14$)

Sol. y = 0.01 m sin (20 πx) cos 200 πt



no. of nodes is 6

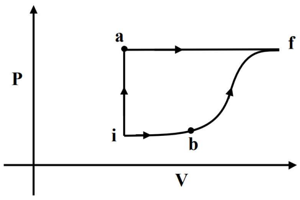
$20\pi = \frac{2\pi}{\lambda}$

$\therefore \quad \lambda = \frac{1}{10}\text{m} = 0.1\text{ m}$

length of the string $2.5\lambda = 0.25m$, Mid point is the antinode

n = 6, ℓ = 0.25m and p = 0.01m

9. A thermodynamic system is taken from an initial state i with internal energy U_i = 100 J to the final state f along two different paths iaf and ibf, as schematically shown in the figure. The work done by the system along the paths af, ib and bf are W_{af} = 200 J, W_{ib} = 50 J and W_{bf} = 100 J respectively. The heat supplied to the system along the path iaf, ib and bf are Q_{iaf}, Q_{ib} and Q_{bf} respectively. If the internal energy of the ystem in the state b is U_b = 200 J and Q_{iaf} = 500 J. Calculate Q_{bf} - Q_{ib} = _____J.



Sol. $U_b = 200$ J, $U_i = 100$ J

Process iaf

Process	W(in Joule)	ΔU(in Joule)	Q(in Joule)
ia		0	
af		200	
Net	300	200	500

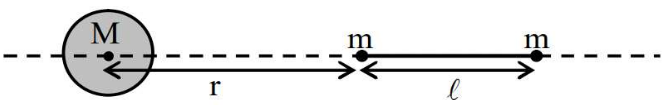
$\Rightarrow$ U_f = 400 Joule

Process ibf

Process	W(in Joule)	ΔU(in Joule)	Q(in Joule)
ib	100	50	150
bf	200	100	300
Net	300	150	450

$Q_{bf} - Q_{ib} = 300 - 150 = 150J$

10. A large spherical mass M is fixed at one position and two identical point masses m are kept on a line passing through the centre of M (see figure). The point masses are connected by a rigid massless rod of length ℓ and this assembly is free to move along the line connecting them. All three masses interact only through their mutual gravitational interaction. When the point mass nearer to M is at a distance r = 3 ℓ from M, the tension in the rod is zero for $m = k\left(\frac{M}{288}\right)$ .The value of k is



Sol. For m closer to M

$$\frac{GMm}{9\ell^2} - \frac{Gm^2}{\ell^2} = ma \quad \text{...(i)}$$

and for the other m :

$$\frac{Gm^2}{\ell^2} + \frac{GMm}{16\ell^2} = ma \quad \text{...(ii)}$$

From both the equations,
k = 7

11. The densities of two solid spheres A and B of the same radii R vary with radial distance r as $\rho_A(r) = k\left(\frac{r}{R}\right)$ and $\rho_B(r) = k\left(\frac{r}{R}\right)^5$, respectively, where k is a constant. The moments of inertia of the individual spheres about axes passing through their centres are I_A and I_B, respectively. If $\frac{I_B}{I_A} = \frac{n}{10}$, the value of n is ____.

Sol.

$$I = \int \frac{2}{3}\rho 4\pi r^2 r^2 dr$$

$$I_A \propto \int (r)(r^2)(r^2)\,dr$$

$$I_B \propto \int (r^5)(r^2)(r^2)dr$$

$$\therefore \frac{I_B}{I_A} = \frac{6}{10}$$

12. A block with mass M = 0.81 kg is connected by a massless spring with stiffness constant k = 10N/m to a rigid wall and moves without friction on a horizontal surface. The block oscillates with small amplitude A about an equilibrium position x_0. Consider two cases: (i) when the block is at x_0 ; and (ii) when the block is at $x = x_0 + A$. In both the cases, a particle with mass m = 0.09 kg is softly placed on the block after which they stick to each other. If A_i and A_{ii} be the amplitudes of oscillation in first and second case respectively. Calculate $10\left(\frac{A_i}{A_{ii}}\right)^2 =$ _____.

Sol.

$$\omega' = \sqrt{\frac{k}{M+m}}$$

$$MA\sqrt{\frac{k}{M}} = (M+m)A'\sqrt{\frac{k}{M+m}}, \text{ so } A' = A\sqrt{\frac{M}{M+m}}$$

in case ii amplitude remains unchanged $\frac{A_i}{A_{ii}} = \sqrt{\frac{M}{M+m}} = \sqrt{\frac{81}{90}} \Rightarrow \left(\frac{A_i}{A_{ii}}\right)^2 = \frac{9}{10}$

13. Consider two solid spheres P and Q each of density 8 gm cm^{-3} and diameters 1cm and 0.5cm, respectively. Sphere P is dropped into a liquid of density 0.8 gm cm^{-3} and viscosity $\eta = 3$ poiseulles. Sphere Q is dropped into a liquid of density 1.6 gm cm^{-3} and viscosity $\eta = 2$ poiseulles. The ratio of the terminal velocities of P and Q is ____.

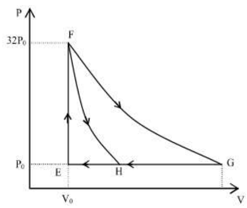
Sol. Terminal velocity $v_T = \frac{2}{9}\frac{r^2}{\eta}(\rho - \sigma)g$, where ρ is the density of the solid sphere and σ is the density of the liquid

$$\therefore \quad \frac{v_P}{v_Q} = \frac{(8-0.8)\times\left(\frac{1}{2}\right)^2 \times 2}{(8-1.6)\times\left(\frac{1}{4}\right)^2 \times 3} = 3$$

SECTION 4 (Maximum Marks: 12)

- This section contains FOUR (04) Matching List Sets.
- Each set has ONE Multiple Choice Question.
- Each set has TWO lists: List-I and List-II.
- List-I has Four entries (P), (Q), (R) and (S) and List-II has Five entries (1), (2), (3), (4) and (5).
- FOUR options are given in each Multiple Choice Question based on List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 ONLY if the option corresponding to the correct combination is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : -1 In all other cases.

14. One mole of mono-atomic ideal gas is taken along two cyclic processes E→F→G→E and E→F→H→E as shown in the PV diagram. The processes involved are purely isochoric, isobaric, isothermal or adiabatic.



Match the paths in List I with the magnitudes of the work done in List II and select the correct answer using the codes given below the lists.

	List I		List II
P.	G → E	1.	160 P_0V_0 ln2
Q.	G → H	2.	36 P_0V_0
R.	F → H	3.	24 P_0V_0
S.	F → G	4.	31 P_0V_0

(A) P. → (1); Q. → (3); R. → (2); S. → (4)
(B) P. → (4); Q. → (3); R. → (2); S. → (1)
(C) P. → (2); Q. → (4); R. → (1); S. → (3)
(D) P. → (4); Q. → (2); R. → (1); S. → (3)

Sol. P. → (4) ; Q. → (3); R. → (2); S. → (1)

Apply $PV^{1+2/3}$ = constant for F to H.

$(32P_0)V_0^{5/3} = P_0 V_H^{5/3} \Rightarrow V_H = 8V_0$

For path FG PV = constant

$\Rightarrow (32P_0)V_0 = P_0V_G \Rightarrow V_G = 32V_0$

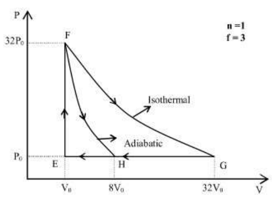
Work done in GE = 31 P_0V_0

Work done in GH = 24 P_0V_0

Work done in FH = $\frac{P_HV_H - P_FV_F}{(-2/f)} = 36P_0V_0$

Work done in FG = $RT \ln\left(\frac{V_G}{V_F}\right)$

$= 160P_0V_0 \ln 2$.



15. A person in a lift is holding a water jar, which has a small hole at the lower end of its side. When the lift is at rest, the water jet coming out of the hole hits the floor of the lift at a distance d of 1.2 m from the person. In the following, state of the lift's motion is given in List I and the distance where the water jet hits the floor of the lift is given in List II. Match the statements from List I with those in List II and select the correct answer using the code given below the lists.

List I	List II
P. Lift is accelerating vertically up.	1. $d = 1.2$ m
Q. Lift is accelerating vertically down with an acceleration less than the gravitational acceleration.	2. $d > 1.2$ m
R. Lift is moving vertically up with constant speed.	3. $d < 1.2$ m
S. Lift is falling freely.	4. No water leaks out of the jar

(A) P-2, Q-3, R-2, S-4 (B) P-2, Q-3, R-1, S-4
(C) P-1, Q-1, R-1, S-4 (D) P-2, Q-3, R-1, S-1

Sol. In P, Q, R no horizontal velocity is imparted to falling water, so d remains same.
In S, since its free fall, $a_{eff} = 0$
Liquid won't fall with respect to lift.

16. A planet of mass M, has two natural satellites with masses m_1 and m_2. The radii of their circular orbits are R_1 and R_2 respectively. Ignore the gravitational force between the satellites. Define v_1, L_1, K_1 and T_1 to be, respectively, the orbital speed, angular momentum, kinetic energy and time period of revolution of satellite 1; and v_2, L_2, K_2 and T_2 to be the corresponding quantities of satellite 2. Given $m_1/m_2 = 2$ and $R_1/R_2 = 1/4$, match the ratios in List-I to the numbers in List-II.

LIST–I	LIST–II
P. $\frac{v_1}{v_2}$	**1.** $\frac{1}{8}$
Q. $\frac{L_1}{L_2}$	**2.** 1
R. $\frac{K_1}{K_2}$	**3.** 2
S. $\frac{T_1}{T_2}$	**4.** 8

(A) **P → 4; Q → 2; R → 1; S → 3**
(B) **P → 3; Q → 2; R → 4; S → 1**
(C) **P → 2; Q → 3; R → 1; S → 4**
(D) **P → 2; Q → 3; R → 4; S → 1**

Sol. **B**

$v \propto \frac{1}{\sqrt{r}}$, Hence correct answer is 'B'

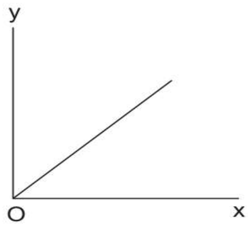
Also, $T \propto r^{3/2}$

$K \propto \frac{m}{r}$

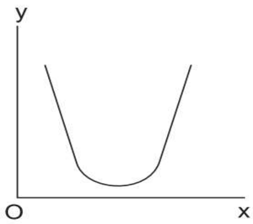
17. Column A gives a list of possible set of parameters measured in some experiments. The variations of the parameters in the form of graphs are shown in column B.

Column A — Column B

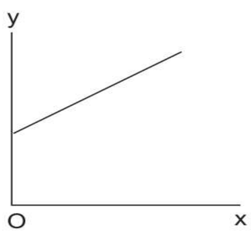
(i) The potential energy of a simple pendulum (y-axis) as a function of its displacement (x-axis)

(a)


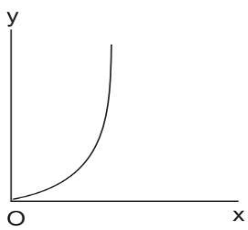
(ii) Displacement (y-axis) as a function of time (x-axis) for a one-dimensional motion at zero or constant acceleration when the body is moving along the positive x-direction

(b)


(iii) The range of a projectile (y-axis) as a function of its velocity (x-axis) when projected at a fixed angle

(c)


(iv) The square of the time period (y-axis) of a simple pendulum as a function of its length (x-axis)

(d)


(A) (i) $\rightarrow$ a,d (ii) $\rightarrow$ b,d (iii) $\rightarrow$ d (iv) $\rightarrow$ b
(B) (i) $\rightarrow$ a (ii) $\rightarrow$ b,d (iii) $\rightarrow$ b (iv) $\rightarrow$ d
(C) (i) $\rightarrow$ d (ii) $\rightarrow$ a,d (iii) $\rightarrow$ d (iv) $\rightarrow$ b
(D) (i) $\rightarrow$ b (ii) $\rightarrow$ b,d (iii) $\rightarrow$ d (iv) $\rightarrow$ a,d

CHEMISTRY

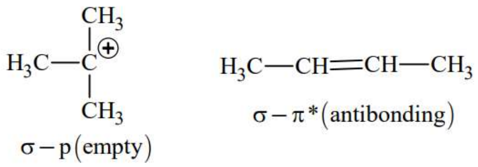
SECTION 1 (Maximum Marks: 12)

- This section contains THREE (03) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 ONLY if (all) the correct option(s) is(are) chosen;
  Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;

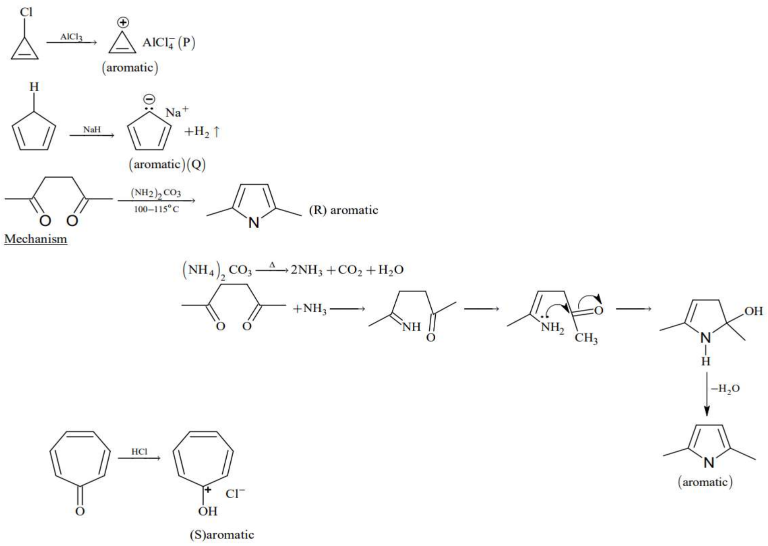
Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;
Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : -2 In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
choosing ONLY (A), (B) and (D) will get +4 marks;
choosing ONLY (A) and (B) will get +2 marks;
choosing ONLY (A) and (D) will get +2 marks;
choosing ONLY (B) and (D) will get +2 marks;
choosing ONLY (A) will get +1 mark;
choosing ONLY (B) will get +1 mark;
choosing ONLY (D) will get +1 mark;
choosing no option (i.e. the question is unanswered) will get 0 marks; and choosing any other combination of options will get -2 marks.

1. Sol. **(A)**



2. Sol.



3. Sol. pKa of PhOH (carbolic acid) is 9.98 and that of carbonic acid (H_2CO_3) is 6.63 thus phenol does not give effervescence with HCO_3^- ion.

SECTION 2 (Maximum Marks: 12)

- This section contains FOUR (04) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : -1 In all other cases.

4. **(B)**

$$4HNO_3 \longrightarrow 2H_2O + 4NO_2 + O_2$$

NO_2 remains dissolved in nitric acid colouring it yellow or even red at higher temperature.

5. **(C)**

Combustion of glucose

$C_6H_{12}O_6 + 6O_2 \longrightarrow 6CO_2 + 6H_2O$

$\Delta H_{combustion} = (6 \times \Delta H_f CO_2 + 6 \times \Delta H_f H_2O) - \Delta H_f C_6H_{12}O_6$

= (6 × - 400 + 6 × - 300) – (- 1300)

= - 2900 kJ/mol

= - 2900/180 kJ/g

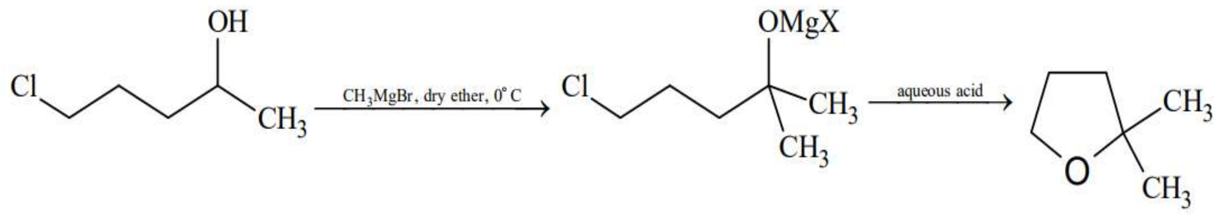
= - 16.11 kJ/g

Hence (C) is correct.

6. III > II > I

More the branching in an alkane, lesser will be the surface area, lesser will be the boiling point.

7.



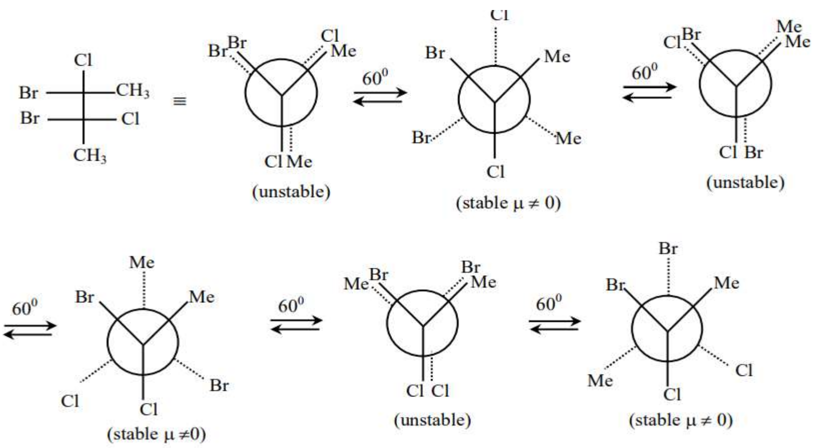
SECTION 3 (Maximum Marks: 24)

- This section contains SIX (06) questions.
- The answer to each question is a NON-NEGATIVE INTEGER.
- For each question, enter the correct integer corresponding to the answer on OMR sheet.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 If ONLY the correct integer is entered;

  Zero Marks : 0 In all other cases.

8.



9. $n = 4$

$\ell = 0, 1, 2, 3$

$|m_\ell| = 1 \Rightarrow \pm 1$

$m_s = -\frac{1}{2}$

For $\ell = 0,\ m_\ell = 0$

$\ell = 1,\ m_\ell = -1,\ 0,\ +1$

$\ell = 2,\ m_\ell = -2, -1, 0, +1, +2$

$\ell = 3,\ m_\ell = -3, -2, -1, 0, +1, +2, +3$

So, six electrons can have $|m_\ell| = 1$ & $m_s = -\frac{1}{2}$ **37.** $k = \frac{R}{N_A}$

$R = kN_A$

$= 1.380 \times 10^{-23} \times 6.023 \times 10^{23}$

$= 8.31174$

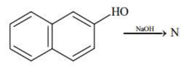
≈ 8.312

10. Here, $V_{solution} \approx V_{solvent}$

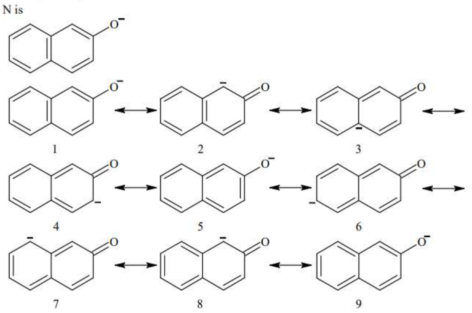
Since, in 1ℓ solution, 3.2 moles of solute are present,

So, 1ℓ solution $\approx$ 1ℓ solvent ($d = 0.4$g/ml) $\approx$ 0.4 kg

So, molality $(m) = \frac{\text{moles of solute}}{\text{mass of solvent (kg)}} = \frac{3.2}{0.4} = 8$

11. 

N is



12. $BeCl_2$, N_3^-, N_2O, NO_2^+, O_3, SCl_2, ICl_2^-, I_3^-, XeF_2

$BeCl_2 \longrightarrow sp \longrightarrow$ linear

$N_3^- \longrightarrow sp \longrightarrow$ linear

$N_2O \longrightarrow sp \longrightarrow$ linear

$\overset{\oplus}{N}O_2 \longrightarrow sp \longrightarrow$ linear

$O_3 \longrightarrow sp^2 \longrightarrow$ bent

$SCl_2 \longrightarrow sp^3 \longrightarrow$ bent

$I_3^- \longrightarrow sp^3d \longrightarrow$ linear

$ICl_2^- \longrightarrow sp^3d \longrightarrow$ linear

$XeF_2 \longrightarrow sp^3d \longrightarrow$ linear

So among the following only four (4) has linear shape and no d-orbital is involved in hybridization.

13. **(9)**

$$m = \frac{X_A \times 1000}{X_B \times M_A}$$

$$m = \frac{1000}{9M_A} \quad \ldots \text{(i)}$$

$$M = \frac{n_B \times 1000 \times d}{n_A \times M_A + n_B \times M_B} = \frac{X_B \times 1000 \times d}{X_A \times M_A + X_B \times M_B}$$

$$= \frac{200}{0.9M_A + 0.1M_B}$$

$$= \frac{2000}{9M_A + M_B} \quad \ldots \text{(ii)}$$

As $m = M$

$$\frac{1000}{9M_A} = \frac{2000}{9M_A + M_B}$$

$$9M_A + M_B = 18M_A$$

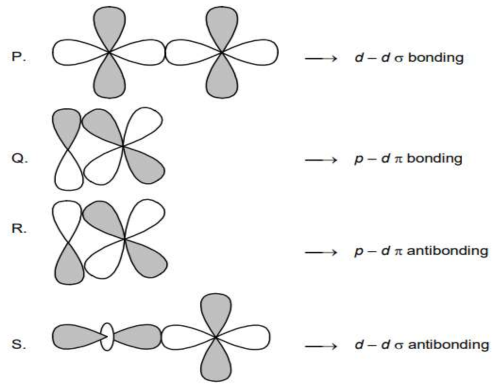
$$\therefore \quad 9M_A = M_B$$

$$\therefore \quad \frac{M_B}{M_A} = 9$$

SECTION 4 (Maximum Marks: 12)

- This section contains FOUR (04) Matching List Sets.
- Each set has ONE Multiple Choice Question.
- Each set has TWO lists: List-I and List-II.
- List-I has Four entries (P), (Q), (R) and (S) and List-II has Five entries (1), (2), (3), (4) and (5).
- FOUR options are given in each Multiple Choice Question based on List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 ONLY if the option corresponding to the correct combination is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : -1 In all other cases.

14.



15. **(D)** (A) $\rightarrow$ (R, T); (B) $\rightarrow$ (P, Q, S); (C) $\rightarrow$ (P, Q, S); (D) $\rightarrow$ (P, Q, S, T)

16. **D**

For H-atom:

$$\underset{(1)}{\text{1s orbital}} - \Psi_{n\ell m} \propto \underset{(i)}{\left(\frac{Z}{a_0}\right)^{3/2}} e^{-\left(\frac{Zr}{a_0}\right)}, \text{S}$$

$$E_4 - E_2 = -\frac{13.6}{16} - \left(-\frac{13.6}{4}\right) = \frac{3 \times 13.6}{16}$$

$$E_6 - E_2 = -\frac{13.6}{36} - \left(-\frac{13.6}{4}\right) = \frac{8 \times 13.6}{36}$$

$E_4 - E_2$ is $\frac{27}{32}$ times of $E_6 - E_2$

17. **D**

$$\frac{K_w}{K_a} = \frac{[CH_3COO^-][OH^-]}{[CH_3COOH]}$$

$$[OH^-] = \left(\frac{K_w}{K_a} \times C\right)^{1/2}$$

(P) is a buffer, so $[H^+]$ does not change on dilution, as [salt] = [acid].

(Q) contains only CH_3COONa

So $CH_3COO^- + H_2O \rightleftharpoons CH_3COOH + OH^-$

$[OH^-] = \sqrt{K_h \times C} \Rightarrow [H^+]$ decreases by $\sqrt{2}$ times

(R) is also salt hydrolysis

So, $NH_4^+ + H_2O \rightleftharpoons NH_4OH + H^+$

$$[H^+] = \sqrt{\frac{K_w}{K_b} \times C}$$

So, C is made $\frac{1}{2}$ so, $[H^+]$ becomes $\frac{1}{\sqrt{2}}$

(S) it is a solubility equilibria

So dilution does not effect $[H^+]$ or $[OH^-]$

MATHEMATICS

SECTION 1 (Maximum Marks: 12)

- This section contains THREE (03) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 ONLY if (all) the correct option(s) is(are) chosen;
  Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;
  Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;
  Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : -2 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks; choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks; choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark; choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option (i.e. the question is unanswered) will get 0 marks; and choosing any other combination of options will get -2 marks.

1. If $ax^2 + bx + c = 0$ and $cx^2 + bx + a = 0$ $(a, b, c \in R)$ have a common non-real root, then

(A) $-2|a| < b < 2|a|$ **(B)** $-2|c| < |b| < 2|c|$ (C) $a = \pm c$ **(D)** $a = c$

Sol. If two quadratic eq^n has one non real root common, then both roots must be common.

$\Rightarrow \frac{a}{c} = \frac{b}{b} = \frac{c}{a} \Rightarrow \boxed{a = c}$ (a, b, d)

$D < 0 \Rightarrow b^2 < 4ac$

$\Rightarrow b^2 < 4a^2$ or $b^2 < 4c^2$

$\Rightarrow -2|a| < b < 2|a|$ or $-2|c| < b < 2|c|$

2. Let $a_n = \underbrace{(111....1)}_{n\ times}$, then

(A) a_{912} is not prime **(B)** a_{951} is not prime **(C)** a_{480} is not prime **(D)** a_{91} is not prime

Sol. $a_n = \underbrace{(111----1)}_{n\ times}$

$a_n = \frac{1}{9}\underbrace{(999----9)}_{n\ times} = \frac{10^n - 1}{10 - 1}$

Now if n is not prime

$\Rightarrow n = p \cdot q$

$a_n = \frac{10^{pq} - 1}{10 - 1} = \frac{(10^p)^q - 1}{10 - 1}$

Let $10^p = t$

$a_n = \frac{t^q - 1}{t - 1} \cdot \frac{t - 1}{10 - 1}$

$= \frac{t^q - 1}{t - 1} \cdot \frac{10^p - 1}{10 - 1}$

$= (1 + t + t^2 + --- t^{n-1})(1 + 10 + 10^2 + --- 10^{p-1})$

$\Rightarrow$ if n is not a prime no., then a_n can be factorized (a, b, c, d)

3. If $\tan x = \frac{2b}{a-c}, (a \neq c)$ y = a $\cos^2 x$ + 2b $\sin x \cos x$ + c $\sin^2 x$, z = a $\sin^2 x$ - 2b $\sin x \cos x$ + c $\cos^2 x$, then

(A) y = z **(B)** y + z = a + c **(C)** y - z = a - c (D) y - z = $(a - c)^2 + 4b^2$

Sol. $\tan x = \frac{2b}{a-c}$, $a \neq c$

$y = a\cos^2 x + 2b\sin x\cos x + c\sin^2 x$ ....(i)

$z = a\sin^2 x - 2b\sin x\cos x + c\cos^2 x$ ....(ii)

adding (i) & (ii)

$(y+z) = a + c \rightarrow$ (B)

Substracting (i) & (ii)

$(y-z) = (a-c)\cos^2 x - (a-c)\sin^2 x + 4b\sin x\cos x$

$\Rightarrow (y-z) = (a-c)(\cos^2 x - \sin^2 x) + 2b\sin 2x$

$\Rightarrow y - z = (a-c)\left[1 - 2\sin^2 x + \left(\frac{2b}{a-c}\right)2\sin x\cos x\right]$

$\Rightarrow y - z = (a-c)[1 - 2\sin^2 x + \tan x\, 2\sin x\cos x]$

$\Rightarrow y - z = (a-c)[1 - 2\sin^2 x + 2\sin^2 x]$

$\Rightarrow y - z = a - c \rightarrow$ (C)

SECTION 2 (Maximum Marks: 12)

- This section contains FOUR (04) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 If ONLY the correct option is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : -1 In all other cases.

4. If $a > 0$ and the equation $|z - a^2| + |z - 2a| = 3$ represents an ellipse then a lies in

(A) (1, 3) (B) $(\sqrt{2}, \sqrt{3})$ **(C)** (0, 3) (D) $(1, \sqrt{3})$

Sol. $|z - z_1| + |z - z_2| = K$

represents an ellipse if

$K > |z_1 - z_2|$

$|z - a^2| + |z - 2a| = 3$ $a > 0$ — (i)

will represent ellipse if

$|a^2 - 2a| < 3$

$\Rightarrow a^2 - 2a > -3$ or $a^2 - 2a < 3$

$\Rightarrow a^2 - 2a + 3 > 0$ or $a^2 - 2a - 3 < 0$

$D < 0$ $(a+1)(a-3) < 0$

$\Rightarrow a \in R$ —(ii) $a \in (-1, 3)$ -- —(iii)

from (i), (ii) & (iii) $a \in (0, 3)$

5. If the product of the perpendicular distances from any point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ of eccentricity $e = \sqrt{3}$ on its asymptotes is equal to 6, then the length of the transverse axis of the hyperbola is

(A) 3 (B) 6 (C) 8 (D) 12

Sol. Let any point on hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

is $P(a\sec\theta, b\tan\theta)$

Assymptotes: $y = \pm \frac{b}{a}x$;

$\Rightarrow ay + bx = 0$ & $ay - bx = 0$

Product of perpendicular $= P_1 \cdot P_2$

$P_1 = \left|\frac{ab\tan\theta + ab\sec\theta}{\sqrt{a^2+b^2}}\right|$, $P_2 = \left|\frac{ab\tan\theta - ab\sec\theta}{\sqrt{a^2+b^2}}\right|$

$P_1 P_2 = \frac{ab}{a^2+b^2} = 6$ —(1)

6. If $w = \frac{z}{z - \frac{1}{3}i}$ and $|w| = 1$, then z lies on

(A) a parabola (**B**) a straight line (C) a circle (D) an ellipse

Sol. $\omega = \frac{z}{z - \frac{1}{3}i}$

$|\omega| = \frac{|z|}{|z - \frac{1}{3}i|}$

$|z| = \left|z - \frac{1}{3}i\right|$

$x^2 + y^2 = x^2 + \left|y - \frac{1}{3}\right|^2$

$y - \frac{1}{3} = \pm y$ Two st. lines

7. If the two circles, $x^2 + y^2 + 2g_1x + 2f_1y = 0$ and $x^2 + y^2 + 2g_2x + 2f_2y = 0$ touches each other, then

(A) $f_1g_1 = f_2g_2$ (B) $\frac{f_1}{g_1} = \frac{t_2}{g_2}$ (C) $f_1f_2 = g_1g_2$ (D) None of these

Sol. $x^2 + y^2 + 2g_1x + 2f_1y = 0$

C_1 : centre : $(-g_1, -f_1)$, $r_1 = \sqrt{g_1^2 + f_1^2}$

$x^2 + y^2 + 2g_2x + 2f_2y = 0$

centre $C_2 : (-g_2, -f_2)$, $r_2 = \sqrt{g_2^2 + f_2^2}$

if two circle touch each other,

$C_1C_2 = |r_1 \pm r_2|$

$\Rightarrow \sqrt{(g_1 - g_2)^2 + (f_1 - f_2)^2} = \left|\sqrt{g_1^2 + f_1^2} \pm \sqrt{g_2^2 + f_2^2}\right|$

After simplification $g_1f_2 = f_1g_2$

$\Rightarrow \frac{f_1}{g_1} = \frac{f_2}{g_2}$.

SECTION 3 (Maximum Marks: 24)

- This section contains SIX (06) questions.
- The answer to each question is a NON-NEGATIVE INTEGER.
- For each question, enter the correct integer corresponding to the answer on OMR sheet.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 If ONLY the correct integer is entered;
  Zero Marks : 0 In all other cases.

8. Let K is a positive integer such that $36 + K, 300 + K, 596 + K$ are the squares of three consecutive terms of an arithmetic progression. Find (K-920).

Sol. Let numbers are $a-d$, a, $a+d$

$(a-d)^2 = 36+k$, — (i)

$(a+d)^2 = 596+k$ — (ii)

$a^2 = 300+k$ — (iii)

(i) + (ii) $\Rightarrow a^2+d^2 = 316+k$ — (iv)

from (iii) & (iv)

$d^2 = 16$ — (v)

(ii) − (i) $\Rightarrow 4ad = 560$

$ad = 140$

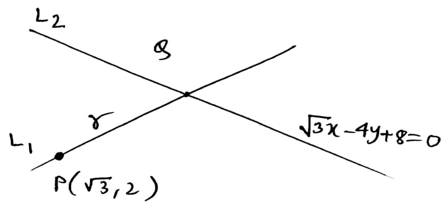
$a^2 = \frac{140 \times 140}{16}$ (from v)

$\Rightarrow a^2 = 1225$

from (iii) $k = a^2 - 300 = 925$

$\Rightarrow$ $\boxed{k - 920 = 5}$

9. If the straight line drawn through the point $P(\sqrt{3}, 2)$ and inclined at an angle $\frac{\pi}{6}$ with the x-axis, meets the line $\sqrt{3}x - 4y + 8 = 0$ at Q. Find the length PQ



Sol. Let PQ = r

Then parametric co-ordinate of Q is

$Q \equiv (\sqrt{3}+|r|\cos\theta, 2+|r|\sin\theta)$.

Now Q lies on L_2.

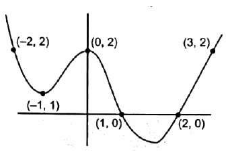
$\Rightarrow \sqrt{3}(\sqrt{3}+|r|\cos\theta) - 4(2+|r|\sin\theta) + 8 = 0$

$\because \theta = \pi/6$

$\Rightarrow 3 + \frac{3|r|}{2} - 8 - 2|r| + 8 = 0$

$\Rightarrow \frac{|r|}{2} = 3 \Rightarrow |r| = 6$.

10. In the given figure, the graph of $y = p(x) = x^4 + ax^3 + bx^2 + cx + d$ go is given



The product of all the imaginary roots of $p(x) = 0$ is

Sol. Given,

$p(x) = x^4 + ax^3 + bx^2 + cx + d$

It is passing through $(1, 0)$

Hence, $p(1) = 0$

$\Rightarrow a + b + c + d = 0$

It is passing through $(0, 2)$. Hence, we get,

$\Rightarrow p(0) = 2$

$\Rightarrow d = 2$

The product of all the roots of the bi-quadratic equation is given as,

a constant term/coefficient of $x^4 = 2$

From the graph given, we can see that the real roots of $p(x)$ are given as,

$x = 1, 2$ (as it intersects the real axis at $(1, 0)$ and $(2, 0)$)

Thus, the product of the real roots is 2.

Hence, the product of the imaginary roots can be given as,

Product of all the roots/Product of real roots.

$= \frac{2}{2}$

$= 1$

11. Let a, b and e be the three distinct non-zero real numbers satisfying the system of equations $\frac{1}{a}+\frac{1}{a-1}+\frac{1}{a-2}=1, \frac{1}{b}+\frac{1}{b-1}+\frac{1}{b-2}=1$ and $\frac{1}{c}+\frac{1}{c-1}+\frac{1}{c-2}=1$. Then abc is equal to

Sol. **Given,**

$\frac{1}{a}+\frac{1}{a-1}+\frac{1}{a-2}=1$

$\frac{1}{b}+\frac{1}{b-1}+\frac{1}{b-2}=1$

$\frac{1}{c}+\frac{1}{c-1}+\frac{1}{c-2}=1$

We can see that a, b and c satisfy the equation of the form given as,

$\frac{1}{x}+\frac{1}{(x-1)}+\frac{1}{x-2}=1$

Thus, the roots of the above equation are a, b and c.

$\Rightarrow x(x-1)(x-2)=(x-1)(x-2)+x(x-1)+x(x-2)$

$x^3-3x^2+2x=3x^2-6x+2$

$x^3-6x^2+8x-2=0$

The product of the roots of abc is 2.

12. Let X_1, X_2, X_3... are in arithmetic progression ession w with a common difference equal to d which is a two digit natural number. Y_1, Y_2, Y_3 ...are in geometric progression with common ratio equal to 16. Arithmetic mean of $X_1, X_2... X_n$ is equal to the arithmetic mean of $Y_1, Y_2 .... Y_n$ which is equal to 5. If the arithmetic mean of $X_6, X_7... X_n + 5$ is equal to the mean arithmetic mean of $Y_{p+1}, Y_{p+2}... Y_{p+n}$, then d is equal to

Sol.

$\text{mean}(X_1, X_2, X_3, ---, X_n) = 5 \quad -(i)$

$\text{Mean}(Y_1, Y_2, Y_3, ----, Y_n) = 5 \quad -(ii).$

$\text{Mean}(X_6, X_7, X_8, --, X_{n+5})$

$= \text{mean}(Y_{1+p}, Y_{2+p}, Y_{3+p}, --, Y_{n+p}).$

$\Rightarrow \text{Mean}\{(X_1+5d), (X_2+5d), ---, (X_n+5d)\}$

$= \text{Mean}\{(Y_1\cdot16^p), (Y_2\cdot16^p), (Y_3\cdot16^p), ---, (Y_n\cdot16^p)\}$

$\Rightarrow \text{Mean}(X_1, X_2, X_3, --X_n) + 5d$

$= 16^p\ \text{mean}(Y_1, Y_2, --- Y_n)$

$\Rightarrow 5+5d = 16^p\cdot 5$

$\Rightarrow d = 16^p - 1.$

Now possible value of $d = 15, 255, ----$

value of d (two digit number) $= 15$

13. If the mean and variance of eight numbers 3, 7, 9, 12, 13, 20, x and y be 10 and 25 respectively, then x . y is equal to

Sol. **Mean**

$$= \overline{x} = \frac{3+7+9+12+13+20+x+y}{8} = 10 \Rightarrow x+y = 16 \ldots \text{(i)}$$

$$\text{Variance } \sigma^2 = \frac{\sum (x_i)^2}{8} - \left(\overline{x}\right)^2 = 25$$

$$\frac{9+49+81+144+169+400+x^2+y^2}{8} - 100 = 25$$

$$\Rightarrow x^2 + y^2 = 148 \ldots \text{(ii)}$$

$$(x+y)^2 = (16)^2 = x^2 + y^2 + 2xy = 256 \Rightarrow xy = 54$$

SECTION 4 (Maximum Marks: 12)

- This section contains FOUR (04) Matching List Sets.
- Each set has ONE Multiple Choice Question.
- Each set has TWO lists: List-I and List-II.
- List-I has Four entries (P), (Q), (R) and (S) and List-II has Five entries (1), (2), (3), (4) and (5).
- FOUR options are given in each Multiple Choice Question based on List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 ONLY if the option corresponding to the correct combination is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : -1 In all other cases.

14. Consider a sequence $\{b_n\}$ of integers such that b_1, b_2, b_3, are in A.P., b_3, b_4, b_5 are in G.P., b_4, b_5, b_6 are in A.P., b_5, b_6, b_7 are in G.P. and so on. Also given that $b_1 = 1$ and $b_5 + b_6 = 198$. Then

Column-I	Column-II
(A) $\sqrt{b_7}$ is equal to	(P) 5
(B) Sum of digits of b_8 is equal to	(Q) 15
(C) $\sqrt{b_9}$ is equal to	(R) 9
(D) Sum of digits of b_{10} is equal to	(S) 17
	(T) 13

(a) A $\rightarrow$ T, B $\rightarrow$ P, C $\rightarrow$ S, D $\rightarrow$ Q (b) A $\rightarrow$ T, B $\rightarrow$ P, C $\rightarrow$ R, D $\rightarrow$ Q
(c) A $\rightarrow$ P, B $\rightarrow$ T, C $\rightarrow$ R, D $\rightarrow$ S (d) A $\rightarrow$ P, B $\rightarrow$ R, C $\rightarrow$ S, D $\rightarrow$ Q

15. Five balls are to be placed in three boxes. Each can hold all the five balls. The number of different ways can we place the balls so that no box remain empty if

Column-I	Column-II
(A) balls and boxes are all different is equal to	(P) 2
(B) balls are identical but boxes are different is equal to	(Q) 6
(C) balls are different but boxes are identical is equal to	(R) 25
(D) balls as well as boxes are identical is equal to	(S) 50
	(T) 150

(a) A $\rightarrow$ P, B $\rightarrow$ Q, C $\rightarrow$ S, D $\rightarrow$ R (b) A $\rightarrow$ T, B $\rightarrow$ Q, C $\rightarrow$ R, D $\rightarrow$ S
(c) A $\rightarrow$ T, B $\rightarrow$ Q, C $\rightarrow$ P, D $\rightarrow$ R (d) A $\rightarrow$ S, B $\rightarrow$ Q, C $\rightarrow$ P, D $\rightarrow$ T

16. Column-I | Column-II

(A) $2^{(32)^{32}}$ is divided by 7, then the remainder is — (P) 0
(B) 5^{99} is divided by 13, then the remainder is — (Q) 2
(C) $(20)^{13}+(13)^{20}$ is divided by 9, then the — (R) 4
(D) $32^{(32)^{32}}$ is divided by 7, then the remainder is — (S) 6
(T) 8

(A) $A\to Q, B\to T, C\to P, D\to R$ (B) $A\to Q, B\to T, C\to S, R\to D$
(C) $A\to R, B\to T, C\to Q, D\to S$ (D) $A\to S, B\to T, C\to P, Q\to R$

17. If Z_1, Z_2, Z_3, Z_4 are the roots of the equation $z^4+z^3+z^2+z+1=0$,then

Column-I | Column-II

(A) $(z_1^2-1)(z_2^2-1)(z_3^2-1)(z_4^2-1)=$ — (P) -1
(B) $(z_1^2+1)(z_2^2+1)(z_3^2+1)(z_4^2+1)=$ — (Q) 0
(C) $z_1^4+z_2^4+z_3^4+z_4^4=$ — (R) 1
(D) The last value of $[|z_1+z_2|]=$ — (S) 4
([.] denote greatest integer function) — (T) 5

(a) $A\to T, B\to R, C\to P, D\to Q$ (b) $A\to P, B\to Q, C\to S, D\to R$
(c) (a) $A\to P, B\to T, C\to Q, D\to R$ (d) $A\to T, B\to R, C\to S, D\to Q$