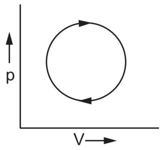
PHYSICS

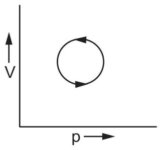
SECTION 1 (Maximum Marks: 12)

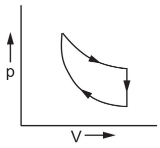
- This section contains FOUR (04) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

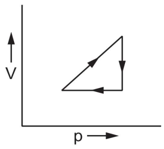
  Full Marks : +3 If ONLY the correct option is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : -1 In all other cases.

1. The following are the p–V diagrams for cyclic processes for a gas. In which of these process heat is released by the gas?

(A) 

(B) 

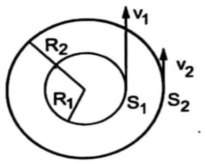
(C) 

(D) 

Sol. In a cyclic process, heat is absorbed by a gas when the work done by it is positive, i.e., when work is done by the gas. Work is done by the gas if the closed curve in a p–V diagram is clockwise, with p on the y-axis and V on the x-axis. If the axes for p and V are interchanged, work done is positive for an anticlockwise curve.

2. Two satellites S_1 and S_2 revolve round a planet in coplanar circular orbits in the same sense. Their periods of revolution are 1 h and 8 h respectively. The radius of the orbit of S_1 is 10 km. When S_2 is closest to S_1, the angular speed of S_2 as observed by an astronaut in S_1 will be

(A) $\frac{\pi}{6}$ rad h^{-1} **(B)** $\frac{\pi}{3}$ rad h^{-1} (C) $\frac{\pi}{4}$ rad h^{-1} (D) $\frac{2\pi}{3}$ rad h^{-1}

Sol. 

$$\left(\frac{R_2}{R_1}\right)^3 = \left(\frac{T_2}{T_1}\right)^2 = \left(\frac{8\text{ h}}{1\text{ h}}\right)^2 = 64$$

or, $\frac{R_2}{R_1} = 4$

or, $R_2 = 4R_1 = 4 \times 10^4$ km.

Now the time period of S_1 is 1 h. So,

$$\frac{2\pi R_1}{v_1} = 1\text{ h}$$

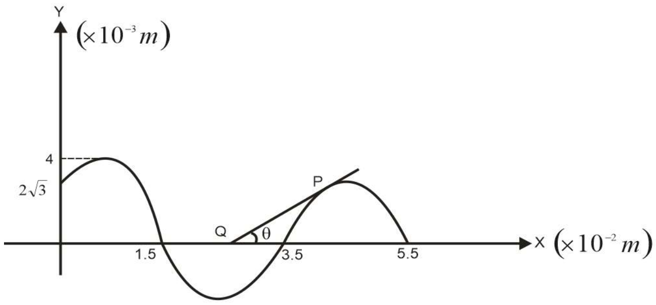
or, $v_1 = \frac{2\pi R_1}{1\text{ h}} = 2\pi \times 10^4 \text{ km h}^{-1}$

similarly, $v_2 = \frac{2\pi R_2}{8h} = \pi \times 10^4 \text{ km h}^{-1}$

$$|v_2 - v_1| = \pi \times 10^4 \text{ km h}^{-1}.$$

$$\omega = \frac{\pi \times 10^4 \text{ km h}^{-1}}{3 \times 10^4 \text{ km}} = \frac{\pi}{3} \text{ rad h}^{-1}.$$

3. A transverse sinusoidal wave is moving along a string in negative x direction with velocity $10\sqrt{3}$ m/s a particle P is observed to be moving up with velocity 10 m/s. A line drawn from the point P intersects the x-axis at point Q. Calculate the angle subtended by line PQ with x-axis. Given that amplitude of the wave is 4 mm, wavelength λ is 0.035 mm



(A) $\frac{\pi}{3}$ **(b)** $\frac{\pi}{6}$ (C) $\frac{\pi}{4}$ (d) 0

Sol. $v_p = -v_w\left(\frac{dy}{dx}\right)$

$10 = -(-10\sqrt{3})\tan\theta \Rightarrow \theta = \frac{\pi}{6}$

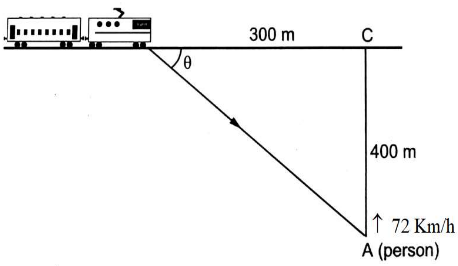
4. A train approaching a railway crossing at a speed of 120 km h^{-1} sounds a short whistle at frequency 640 Hz when it is 300 m away from the crossing. The speed of sound in air is 340 m/s. A person moving on a car with speed 72 Km/h perpendicular to the track through the crossing at a distance of 400 m at the instant when whistle is blown by the train. What will be the frequency heard by the person at that vary instant

(A) 680 Hz (B) 648 Hz **(C)** 712 Hz (D) 650 Hz

Sol. Component of velocity of the source towards listner $u_{s/l} = \frac{100}{3} \times \frac{3}{5} = 20$ m/s

and component of velocity of listner towards source $u_{l/s} = 20 \times \frac{4}{5} = 16$ m/s

$$v' = \frac{v_{sound} + u_{l/s}}{v_{sound} - u_{s/l}} \times v \Rightarrow v = \frac{340+16}{340-20} \times 640 = 712 Hz$$



SECTION 2 (Maximum Marks: 12)

- This section contains THREE (03) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 ONLY if (all) the correct option(s) is(are) chosen;
Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;
Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;
Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks : 0 If unanswered;
Negative Marks : -2 In all other cases.

5. The molar heat capacity for an ideal gas
(a) cannot be negative
(b) must be equal to either C_V or C_p
(c) must lie in the range $C_v \le C \le C_p$
(d) may have any value between $-\infty$ and $+\infty$

Sol. The molar heat capacity has the general definition

$$C = \frac{1}{n} \cdot \frac{\Delta Q}{\Delta T},$$

where n = number of moles, ΔQ = heat absorbed by the gas and ΔT = rise in temperature of gas.

It is possible to obtain almost any set of values for ΔQ and ΔT by proper selection of a process.

6. The stationary waves set up on a string have the equation $y = (2\text{ mm})\sin[(6.28\text{ m}^{-1})x]\cos(\omega t)$. This stationary wave is created by two identical waves, of amplitude A each, moving in opposite directions along the string.
(a) A = 2 mm
(b) A = 1 mm
(c) The smallest length of the string is 50 cm.
(d) The smallest length of the string is 2 m.

Sol. Comparing with the equation $y = 2A\sin\left(\frac{n\pi x}{L}\right)\cos(\omega t)$,

$$2A = 2\text{ mm} \quad \text{or} \quad A = 1\text{ mm}$$

$$\frac{n\pi x}{L} = 6.28x = 2\pi x \quad \text{or} \quad L = \frac{n}{2}\text{ m.}$$

For $n = 1, L = 0.5$ m.

7. A solid sphere of radius R and density ρ is attached to one end of a mass-less spring of force constant k. The other end of the spring is connected to another solid sphere of radius R and density 3ρ. The complete arrangement is placed in a liquid of density 2ρ and is allowed to reach equilibrium. The correct statement(s) is (are)

(A) the net elongation of the spring is $\frac{4\pi R^3 \rho g}{3k}$
(B) the net elongation of the spring is $\frac{8\pi R^3 \rho g}{3k}$
(C) the light sphere is partially submerged.
(D) the light sphere is completely submerged.

Sol. **(A, D)**

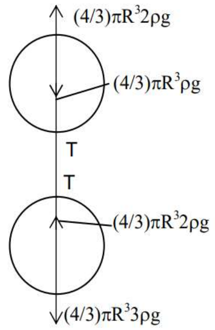
At equilibrium,

$$\frac{4}{3}\pi R^3 2\rho g = \frac{4}{3}\pi R^3 \rho g + T$$

$$T = \frac{4}{3}\pi R^3 \rho g$$

$$\therefore\ \Delta\ell = \frac{4}{3k}\pi R^3 \rho g$$

For equilibrium of the complete system, net force of buoyancy must be equal to the total weight of the sphere which holds true in the given problem. So both the spheres are completely submerged.



SECTION 3 (Maximum Marks: 24)

- This section contains SIX (06) questions.
- The answer to each question is a NON-NEGATIVE INTEGER.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 If ONLY the correct integer is entered;
  Zero Marks : 0 In all other cases.

8. A bob of mass m , suspended by a string of length l_1 is given a minimum velocity required to complete a full circle in the vertical plane. At the highest point, it collides elastically with another bob of mass m suspended by a string of length l_2, which is initially at rest. Both the strings are mass-less and inextensible. If the second bob, after collision acquires the minimum speed required to complete a full circle in the vertical plane, the ratio l_1/l_2 is_____.

Sol. The initial speed of 1st bob (suspended by a string of length l_1)is $\sqrt{5gl_1}$.

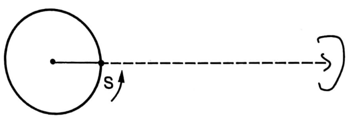
The speed of this bob at highest point will be $\sqrt{gl_1}$.

When this bob collides with the other bob there speeds will be interchanged.

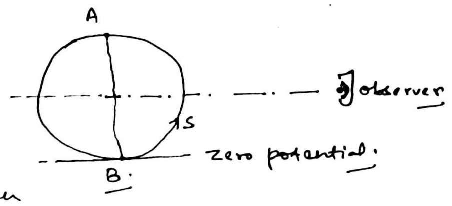
$$\sqrt{gl_1} = \sqrt{5gl_2} \Rightarrow \frac{l_1}{l_2} = 5$$

9. A small source of sound S of frequency 480 Hz is attached to the end of a light string and is whirled in a vertical circle of radius 2 m. The string just remains tight when the source is at the highest point. An observer is located in the same vertical plane at a large distance and at the same height as the centre of the circle as shown . The speed of sound in air = 330 m s^{-1} and g = 10 ms^{-2}. The difference between maximum and minimum frequency heard by the observer will be______Hz (to the nearest integer).

Take $\sqrt{5} = 2.23$



Sol.



max^m frequency will be heard when source passes through 'B' & minimum will be heard when source is passing through 'A'

at point 'A' :- $T + mg = \frac{mv_A^2}{l}$

Since string is just tight $T \to 0$

$V_A = \sqrt{gl} = 2\sqrt{5} \Rightarrow 4.46$ m/sec

$\nu_A = \frac{330}{334.46} \times 480 \Rightarrow 474$ Hz.

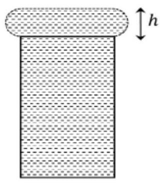
at point 'B' :- $V_B = \sqrt{5gl} \Rightarrow 10$ m/sec

$\nu_B = \frac{330}{320} \times 480 \cdot 1.5$

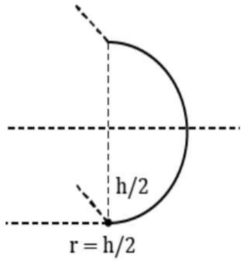
$= 495$ Hz.

$\nu_B - \nu_A = 495 - 474 \Rightarrow 21$ Hz.

10. When water is filled carefully in a glass, one can fill it to a height h above the rim of the glass due to the surface tension of water. To calculate h just before water starts flowing, model the shape of the water above the rim as a disc of thickness h having semicircular edges, as shown schematically in the figure. When the pressure of water at the bottom of this disc exceeds what can be withstood due to the surface tension, the water surface breaks near the rim and water starts flowing from there. If the density of water, its surface tension and the acceleration due to gravity are 10^3 kg m^{-3}, 0.07 Nm^{-1} and 10 ms^{-2}, respectively, the value of h (to the nearest integer) in mm will be _________.



Sol.



$r = \frac{h}{2}$

Pressure at the bottom of disc = pressure due to surface tension

$\rho gh = T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

$R_1 >>> R_2$

So, $\frac{1}{R_1} <<< \frac{1}{R_2}$ and $R_2 = \frac{h}{2}$

$\therefore \rho gh = T\left[\frac{1}{R_1} + \frac{1}{R_2}\right] = T\left[0 + \frac{1}{\frac{h}{2}}\right]$

$h^2 = \frac{2T}{\rho g}$

$h = \sqrt{\frac{2T}{\rho g}} = \sqrt{\frac{2 \times 0.07}{10^3 \times 10}} = \sqrt{\frac{14 \times 100}{10^4 \times 100}}$

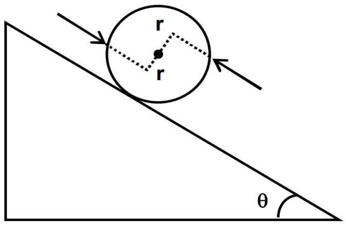
$h = \sqrt{14}$ mm ≈ 3.75mm

11. A piece of ice (heat capacity) 2100 $Jkg^{-1}\,^{\circ}C^{-1}$ and latent heat $= 3.36 \times 10^5\ J\ kg^{-1}$ of mass m grams is at $-5^{\circ}C$ at atmospheric pressure. It is given 420 J of heat so that the ice starts melting. Finally when the ice-water mixture is in equilibrium, it is found that 1 gm of ice has melted. Assuming there is no other heat exchange in the process, the value of m is_____.

Sol. $Q = ms\theta + mL.$

$= 8$

12. A solid sphere of mass 1 kg and radius 1 m rolls without slipping on a fixed inclined plane with an angle of inclination $\theta = 30^{\circ}$ from the horizontal. Two forces of magnitude 1N each, parallel to the incline, act on the sphere, both at distance $r = 0.5$ m from the center of the sphere, as shown in the figure. If a be the acceleration of the sphere down the plane then $14 \times a =$_______ ms^{-2}. (Take $g = 10\ ms^{-2}$.)



Sol. $mg\sin\theta \times R + 1 \times \frac{R}{2} - 1 \times \frac{3R}{2} = I_P\alpha$

$\frac{10}{2} \times 1 + \frac{1}{2} - \frac{3}{2} = \frac{7}{5} \times mR^2\alpha$

$5 - 1 = \frac{7}{3}\alpha$

$\frac{20}{7} = \alpha$

$a_{cm} = R\alpha = \frac{20}{7}$

$14 \times a_{cm} = 40\ m/s^2$

13. A train with cross-sectional area S_t is moving with speed v_t inside a long tunnel of cross-sectional area $S_0 (S_0 = 4S_t)$. Assume that almost all the air (density ρ) in front of the train flows back between its sides and the walls of the tunnel. Also, the air flow with respect to the train is steady and laminar. Take the ambient pressure and that inside the train to be p_0. If the pressure in the region between the sides of the train and the tunnel walls is p, then $p_0 - p = \frac{7}{2N}\rho v_t^2$. The value of N is __________.

Sol. The velocity of air flow relative to the train between its sides and the walls of the tunnel is v

$v \times 3S_t = v_t 4S_t$

$v = \frac{4}{3}v_t$ …(i)

Now,

$P + \frac{1}{2}\rho v^2 = P_0 + \frac{1}{2}\rho v_t^2$

$P_0 - P = \frac{1}{2}\rho(v^2 - v_t^2) = \frac{1}{2}\rho v_t^2\left(\frac{16}{9} - 1\right)$

$P_0 - P = \frac{7}{18}\rho v_t^2$

Hence N = 9

SECTION 4 (Maximum Marks: 12)

- This section contains TWO (02) paragraphs.
- Based on each paragraph, there are TWO (02) questions.
- For each question, enter the correct options on OMR sheet
- Answer to each question will be evaluated according to the following marking scheme:

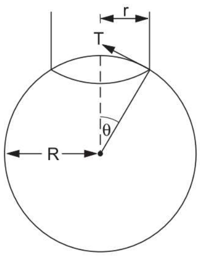
Full Marks : +3 If ONLY the correct option is choosen.
Zero Marks : 0 In all other cases.

When liquid medicine of density $\rho = 10^3$ kg/m^3 is to be put in the eye, it is done with the help of a dropper. As the bulb on the top of the dropper is pressed, a drop forms at the opening of the dropper. We wish to estimate the size of the drop. We first assume that the drop formed at the opening is spherical because that requires a minimum increase in its surface energy. To determine the size, we calculate the net vertical force due to the surface tension T = 0.1 N/m when the radius of the drop is $R = 5\times10^{-3}$ m . When this force becomes smaller than the weight of the drop, the drop gets detached from the dropper.

14. If the radius of the opening of the dropper is $r = 5\times10^{-4}$ m the vertical force due to the surface tension on the drop = ____μN. (Take $\pi = 3.14$)

Sol. Vertical upward force due to surface tension

$= (2\pi r)(T\sin\theta) = 2\pi r T\left(\frac{r}{R}\right) = 2\pi r^2 T/R.$



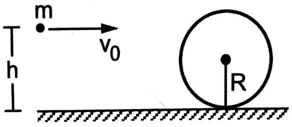
$= 31.4\mu N$

When liquid medicine of density $\rho = 10^3$ kg/m^3 is to be put in the eye, it is done with the help of a dropper. As the bulb on the top of the dropper is pressed, a drop forms at the opening of the dropper. We wish to estimate the size of the drop. We first assume that the drop formed at the opening is spherical because that requires a minimum increase in its surface energy. To determine the size, we calculate the net vertical force due to the surface tension T = 0.1 N/m when the radius of the drop is $R = 5\times10^{-3}$ m . When this force becomes smaller than the weight of the drop, the drop gets detached from the dropper.

15. After the drop detaches, its surface energy is_____μJ.

Sol. Surface energy $= 4\pi R^2 T$
$= 31.4 \mu J$

The sphere of mass M = 15 kg and radius R = 2m shown in figure lies on a rough plane when a particle of mass m=0.015Kg travelling at a speed v_0 = 4 m/s collides and sticks with it. If the line of motion of the particle is at a distance h = 3.2 m above the plane,



16. The angular speed of the system just after the collision = _____

Sol. We shall use conservation of angular momentum about the centre of mass, which is to be taken at the centre of the sphere (M>>m). Angular momentum of the particle before collision is mv_0 (h-R). If the system rotates with angular speed ω after collision, the angular momentum of the system becomes

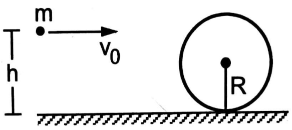
$$mv_0(h-R) = \left(\frac{2}{5}M + m\right)R^2\omega$$

$$\omega = \frac{mv_0(h-R)}{\left(\frac{2}{5}M + m\right)R^2}$$

$$(\text{since } M >> m) \Rightarrow \omega = \frac{mv_0(h-R)}{\left(\frac{2}{5}M\right)R^2}$$

0.003 rad/s

The sphere of mass M = 15 kg and radius R = 2m shown in figure lies on a rough plane when a particle of mass m=0.015Kg travelling at a speed v_0 = 4 m/s collides and sticks with it. If the line of motion of the particle is at a distance h above the plane,



17. The value of h for which the sphere starts pure rolling on the plane = ______.

Sol. The sphere will start rolling just after the collision if $v = \omega R$

$$\frac{mv_0}{M+m} = \frac{mv_0(h-R)}{\left(\frac{2}{5}M + m\right)R}$$

$(\text{since } M >> m)$

$$h=\left(\frac{\frac{7}{5}M+2m}{M+m}\right)R \Rightarrow h \approx \frac{7}{5}R$$

$= 2.8$ m

CHEMISTRY

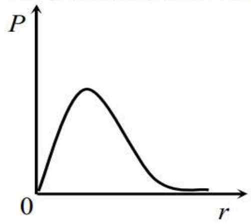
SECTION 1 (Maximum Marks: 12)

- This section contains FOUR (04) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 If ONLY the correct option is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : -1 In all other cases.

1. Which of the following correct order of solubility order of group-I sulphate salt ?
(A) $Li_2SO_4 > Na_2SO_4 > K_2SO_4 > Rb_2SO_4 > Cs_2SO_4$
(B) $Li_2SO_4 < Na_2SO_4 > K_2SO_4 < Rb_2SO_4 < Cs_2SO_4$
(C) $Li_2SO_4 < Na_2SO_4 < K_2SO_4 < Rb_2SO_4 < Cs_2SO_4$
(D) $Li_2SO_4 < Na_2SO_4 < K_2SO_4 > Rb_2SO_4 > Cs_2SO_4$

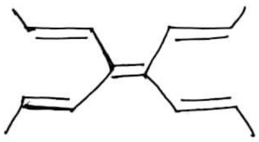
Sol. According to Fajan rule. Size of cation decreases that time polarising power of cation increases, this should lead to increase the covalent character and Solubility decreases. Hence, Li_2SO_4 has maximum covalent character and Cs_2SO_4 maximum ionic character so, solubility order is
=> $Li_2SO_4 < Na_2SO_4 < K_2SO_4 < Rb_2SO_4 < Cs_2SO_4$

2. Sol. **(D)**
The probability distribution curve for 1s electron of hydrogen atom.



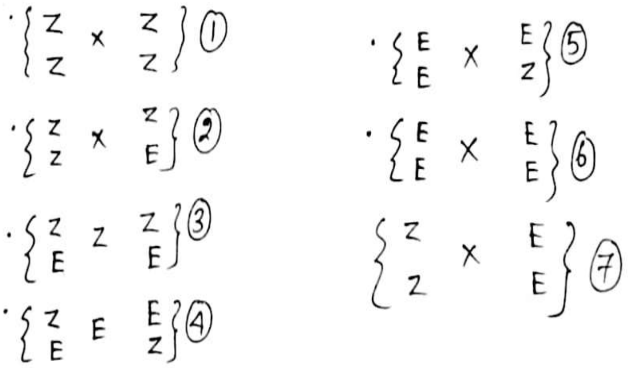
3. Sol. **(B)**
Increasing order of atomic radius of group 13 elements Ga < Al < In < Tl.
Due to poor shielding of d-electrons in Ga, its radius decreases below Al.

4. How many possible geometrical isomers are present of the following compound?



(A) 4 (B) 6 (C) 7 (D) 5

Sol.



SECTION 2 (Maximum Marks: 12)

- This section contains THREE (03) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 ONLY if (all) the correct option(s) is(are) chosen;
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  Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;
  Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;
  Zero Marks : 0 If unanswered;
  Negative Marks : -2 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option(s) (i.e. the question is unanswered) will get 0 marks and
  choosing any other option(s) will get -2 marks.

5. Sol. **A, C**

6. Sol. Since container is thermally insulated. So, q = 0, and it is a case of free expansion therefore W = 0 and $\Delta E = 0$
So, $T_1 = T_2$
Also, $P_1V_1 = P_2V_2$

7. Sol. (A) Ice has cage-like structure in which each water molecule is surrounded by four other water molecules tetrahedrally through hydrogen boding, due to this density of ice is less than water and it floats in water.

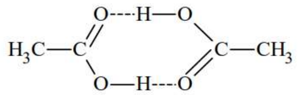
(B) $R-NH_2 + H-OH \rightleftharpoons R-\overset{\oplus}{N}H_3 + OH^-$ (I)

$(R)_3N + H-OH \rightleftharpoons (R)_3-\overset{\oplus}{N}H + OH^-$ (II)

The cation (I) more stabilized through hydrogen boding than cation (II). So, $R–NH_2$ is better base than $(R)_3N$ in aqueous solution.

(C) HCOOH is stronger acid than CH_3COOH due to inductive effect and not due to hydrogen bonding.

(D) Acetic acid dimerizes in benzene through intermolecular hydrogen bonding.



SECTION 3 (Maximum Marks: 24)

- This section contains SIX (06) questions.
- The answer to each question is a NON-NEGATIVE INTEGER.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 If ONLY the correct integer is entered;
  Zero Marks : 0 In all other cases.

8. It has been considered that during the formation of earth H_2 gas was available at the earth. But due to the excessive heat on the earth this has been escaped. The temperature of earth during H_2 gas formation is $A \times 10^{3}$°C. Find the value of A. [Closest integer] [Given : The escaping velocity is 1.1×10^{6}cm/s]

Sol. ⇒ Escape velocity of H_2 should be equal to average velocity of H_2.

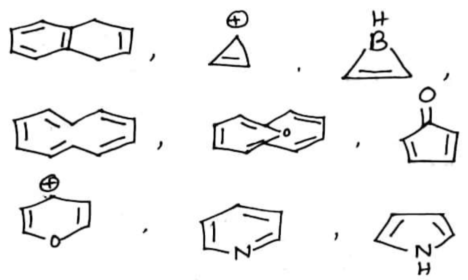
∴ Average velocity of $H_2 = 1.1\times10^{6}$ cm/s $= 1.1\times10^{4}$ m/s.

∴ Average velocity $= \sqrt{\frac{8RT}{\pi M}}$ [$M_{H_2} = 2\times10^{-3}$ kg/mol, $R = 8.314$ J·mol^{-1}K^{-1}, T = ??]

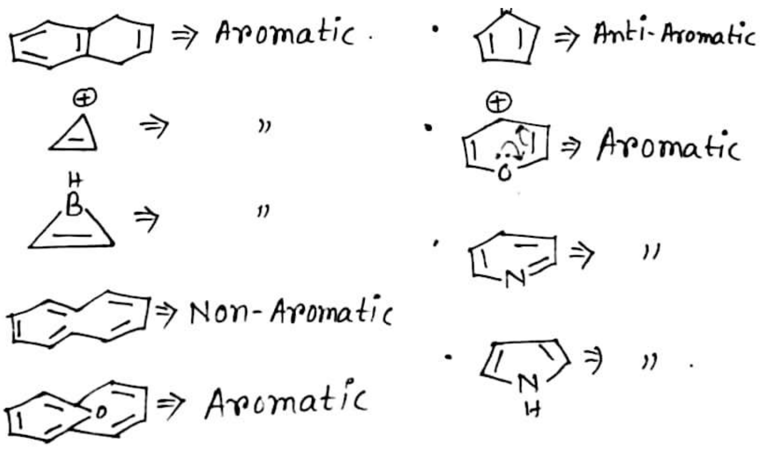
∴ T = 11157.5°C

≃ 11×10^{3}°C.

9. Among the following the number of aromatic compound(s) is_____.



Sol.



10. The solubility of a salt of weak acid (**AB**) at pH 3 is **Y** × 10^{-3} mol L^{-1}. The value of **Y**^{2}× 10^{-6} mol^2 L^{-2} is ____. (Given that the value of solubility product of **AB** (K_{sp}) = 2 × 10^{-10} and the value of ionization constant of **HB** (K_a) = 1 × 10^{-8})

Sol.

$$\underset{}{AB} \rightleftharpoons \underset{S}{A^+} + \underset{(S-x)}{B^-}$$

$$2\times10^{-10} = S(S-x) \quad \ldots (1)$$

$$\underset{(S-x)}{B^-} + \underset{10^{-3}}{H^+} \rightleftharpoons \underset{x}{HB}$$

$$\frac{1}{10^{-8}} = \frac{x}{(S-x)\times10^{-3}}$$

$$\frac{x}{S-x} = 10^5 \quad \ldots (2)$$

Multiply equation (1) and (2).

$S.x = 2 \times 10^{-5}$

From Eq. (1)

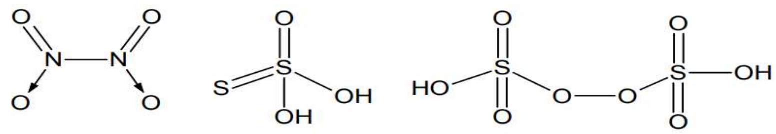
$$S^2 - Sx = 2\times10^{-10}$$

$$S^2 - 2\times10^{-5} = 2\times10^{-10}$$

$$S^2 = 2\times10^{-5} + 2\times10^{-10} \cong 2\times10^{-5}$$

$$S^2 = 20\times10^{-6}$$

11. Sol.



12. The ground state energy of hydrogen atom is – 13.6 eV. Consider an electronic state ψ energy, azumuthal quantum number and magnetic quantum number are -3.4 eV, 2 and 0, respectively. If x is number of angular nodes and y is the number of radial nodes. Calculate x + y = _______.

Sol.

$$-3.4 = \frac{-13.6\times2^2}{n^2}$$

n = 4

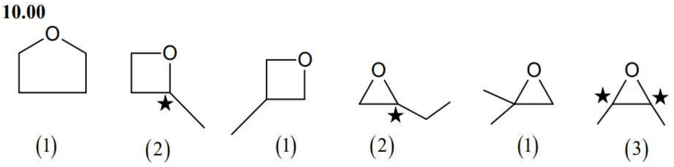
$\ell = 2$

Subshell = 4d

Angular nodes = $\ell = 2$

Radial nodes = $n - \ell - 1 = 4 - 2 - 1 = 1$

13. Sol. **10.00**



SECTION 4 (Maximum Marks: 12)

- This section contains TWO (02) paragraphs.
- Based on each paragraph, there are TWO (02) questions.
- For each question, enter the correct options on OMR sheet
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct option is choosen.

Zero Marks : 0 In all other cases.

14. Sol. $\frac{r_X}{r_Y} = \frac{d}{24-d} = \sqrt{\frac{40}{10}} = 2$

d = 48 – 2d

3d = 48

$d = 16cm$

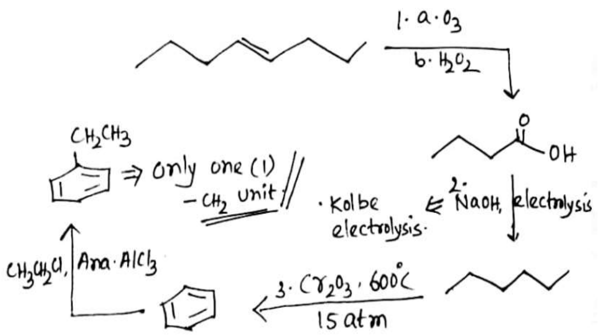
15. Sol. As the collision frequency increases then molecular speed decreases than the expected.

16. 

1. $\xrightarrow[b.H_2O_2]{a.O_3}$
2. NaoH, electrolysis
3. Cr_2O_3, 600ºC, 15 atm
4. CH_3CH_2Cl, Anahydrous $AlCl_3$

The number of – CH_2 – (methylene)
groups in the product formed form the following reaction sequence is _______.

Sol. 1

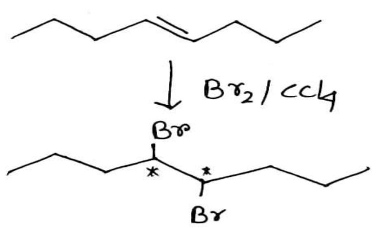


1. $\xrightarrow[b.H_2O_2]{a.O_3}$
2. NaoH, electrolysis
3. Cr_2O_3, 600ºC, 15 atm
4. CH_3CH_2Cl, Anahydrous $AlCl_3$

The number of – CH_2 – (methylene)

17. The total number of chiral center formed from one molecule of compound A on Brominination (Br_2/CCl_4) is ______.

Sol. 2



So, Two(2) chiral center present in product

MATHEMATICS

SECTION 1 (Maximum Marks: 12)

- This section contains FOUR (04) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 If ONLY the correct option is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : -1 In all other cases.

1. An ellipse has eccentricity 1/2 and one focus at point P(1/2,1). Its one directrix nearer to point P is the common tangent, to the circle $x^2 + y^2 = 1$ and the hyperbola $x^2 - y^2 = 1$. The equation of the ellipse is
(A) $3x^2 + 4y^2 - 2x - 8y + 4 = 0$ (B) $3x^2 + 4y^2 + 2x + 8y - 4 = 0$
(C) $3x^2 + 4y^2 - 2x + 8y - 4 = 0$ (4) None of these

Sol. Any point on the hyperbola $x^2 - y^2 = 1$ is $(\sec\theta, \tan\theta)$.

Then tangent at $(\sec\theta, \tan\theta)$ to $x^2 - y^2 = 1$ is

$x\sec\theta - y\tan\theta = 1$ ....(1)

this will also be a tangent to $x^2 + y^2 = 1$ if radius = length of perpendicular from (0,0) to the equation (1)

or $1 = \frac{1}{\sqrt{\sec^2\theta + \tan^2\theta}}$

or $\sec^2\theta + \tan^2\theta = 1$

or $2\tan^2\theta = 0$

or $\tan\theta = 0 \therefore \theta = 0, \pi$

$\therefore$ putting for θ in (1), the common tangents are $\pm x = 1$,

$x = 1 \& x + 1 = 0$

$x = 1$ is nearer to $F\left(\frac{1}{2}, 1\right)$ than $x + 1 = 0$

$\therefore$ the ellipse has the following focus $= \left(\frac{1}{2}, 1\right)$

corresponding directrix is x -1 = 0 and $e = \frac{1}{2}$

$\therefore$ by focus - directirx property, the equation of the ellipse is

$$\sqrt{\left(x - \frac{1}{2}\right)^2 + (y-1)^2} = \frac{1}{2}\left(\frac{x-1}{\sqrt{1^2+0^2}}\right)$$

or $3x^2 + 4y^2 - 2x - 8y + 4 = 0$

2. The value of $\cos\frac{\pi}{65}\cos\frac{2\pi}{65}\cos\frac{4\pi}{65}.....\cos\frac{32\pi}{65}$ is
(A) $\frac{1}{32}$ **(B)** $\frac{1}{64}$ (C) $-\frac{1}{32}$ (D) $-\frac{1}{64}$

Sol.

3. If the points $(-2,0)$, $\left(-1, \frac{1}{\sqrt{3}}\right)$ and $(\cos\theta, \sin\theta)$ are collinear, then the number of values of $\theta \in [0, 2\pi]$ is
(A) 0 (B) 1 (C) 2 (D) infinite

Sol.

4. Let S = $\left\{z \in c : |z-1| = 1 \text{ and } \left(\sqrt{2}-1\right)(z+\bar{z}) - i(z-\bar{z}) = 2\sqrt{2}\right\}$. Let z1, z2 $\in$ S be such that $|z_1| = \max_{z\in S} |z|$ and $|z_2| = \min_{z\in S} |z|$. Then $\left|\sqrt{2}z_1 - z_2\right|^2$ equals :
(A) 1 (B) 4 (C) 3 **(D)** 2

SECTION 2 (Maximum Marks: 12)

- This section contains THREE (03) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).

- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 ONLY if (all) the correct option(s) is(are) chosen;
  Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;
  Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;
  Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;
  Zero Marks : 0 If unanswered;
  Negative Marks : -2 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks; choosing ONLY (A) and (D) will get +2marks;
  choosing ONLY (B) and (D) will get +2 marks; choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark; choosing ONLY (D) will get +1 mark;
  choosing no option(s) (i.e. the question is unanswered) will get 0 marks and
  choosing any other option(s) will get -2 marks.

5. The tangent at any point P on a standard ellipse with foci as S & S' meets the tangents at the vertcies A & A' in the points V & V', then

(A) $\ell(AV).\ell(A'V') = b^2$ (B) $\ell(AV).\ell(A'V') = a^2$

(C) <V'SV = 90C **(D)** VS' VS is a cyclic quadrilateral

6. The equation $x^{\left[(\log_3 x)^2 - \frac{9}{2}(\log_3 x)+5\right]} = 3\sqrt{3}$ has

(A) exactly three real solution (B) at least one real solution

(C) exactly one irrational solution (D) complex roots

7. **A, D**

Sol. $\text{Im}\left(\frac{a(x+iy)+b}{x+iy+1}\right) = y$

$\Rightarrow (x+1)^2 + y^2 = 1$

$\Rightarrow x = -1 \pm \sqrt{1-y^2}$

SECTION 3 (Maximum Marks: 24)

- This section contains SIX (06) questions.
- The answer to each question is a NON-NEGATIVE INTEGER.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 If ONLY the correct integer is entered;
  Zero Marks : 0 In all other cases.

8. If the constant term, in binomial expansion of $\left(2x^r + \frac{1}{x^2}\right)^{10}$ is 180, then r is equal to

Sol.

9. If the line y + x = 0 bisects two chords drawn from a point $\left(\frac{1+a\sqrt{2}}{2}, \frac{1-a\sqrt{2}}{2}\right)$ to the circle $2x^2 + 2y^2 - (1+a\sqrt{2})x - (1-a\sqrt{2})y = 0$, then 'a' lies in the interval $(-\infty, -\lambda) \cup (\lambda, \infty)$, the numerical quantity λ should be equal to

Sol. Equation of chord AB whose mid point is $(h, -h)$ is

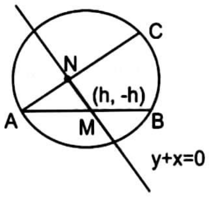
$T = S_1$

$2xh - 2yh - \left(1+\sqrt{2}a\right)\left(\frac{x+h}{2}\right) - \left(1-\sqrt{2}a\right)\left(\frac{y-h}{2}\right)$

$= 2h^2 + 2h^2 - \left(1+\sqrt{2}a\right)h + \left(1-\sqrt{2}a\right)h$

$\Rightarrow 4xh - 4yh - \left(1+\sqrt{2}a\right)(x+h) - \left(1-\sqrt{2}a\right)(y-h)$

$= 8h^2 - 2\left(1+\sqrt{2}a\right)h + 2\left(1-\sqrt{2}a\right)h$



$\Rightarrow x\left[4h - \left(1+\sqrt{2a}\right)\right] - y\left[4h + \left(1-\sqrt{2}a\right)\right] - h\left(1+\sqrt{2}a\right) + h\left(1-\sqrt{2}a\right)$

$= 8h^2 - 2\left(1+\sqrt{2}a\right)h + 2\left(1-\sqrt{2}a\right)h$

or

$8h^2 - \left(1+\sqrt{2}a\right)h + \left(1-\sqrt{2}a\right)h - x\left[4h - \left(1+\sqrt{2}a\right)\right] + y\left[4h + \left(1-\sqrt{2}a\right)\right] = 0$

It passes through $A\left(\frac{1+\sqrt{2}a}{2}, \frac{1-\sqrt{2}a}{2}\right)$ then

$8h^2 - 2\sqrt{2}ah - \left(\frac{1+\sqrt{2}a}{2}\right)\left[4h - \left(1+\sqrt{2}a\right)\right] + \left(\frac{1-\sqrt{2}a}{2}\right)\left[4h + \left(1-\sqrt{2a}\right)\right]$

or $8h^2 - 2\sqrt{2}ah - 2h\left(1+\sqrt{2}a\right) + \frac{\left(1+\sqrt{2}a\right)^2}{2} + 2h\left(1-\sqrt{2}a\right) + \frac{\left(1-\sqrt{2}a\right)^2}{2} = 0$

or $8h^2 - 6\sqrt{2}ah + 1 + 2a^2 = 0$

Hence for two real and different values of h, we must have

$\left(-6\sqrt{2}a\right)^2 - 4 \cdot 8 \cdot (1+2a^2) > 0$

or $a^2 - 4 > 0$

$(a+2)(a-2) > 0$

$\therefore$ $a \in (-\infty, -2) \cup (2, \infty)$

10. From the point (-1, 2) tangent lines are drawn to the parabola $y^2 = 4x$. If area of triangle formed by the chord of contact and the tangents is $\sqrt{2}$ N. then N =

Sol. Let, tangents PA and PB drawn from $P(-1, 2)$ to the parabola $y^2 = 4x$

Equation of the chord of contact AB

$\Rightarrow 2y = 2(x-1)$

$\Rightarrow y = x - 1$

Here, A and B are the points of intersection of $y^2 = 4x$ and $y = x - 1$

$\Rightarrow y^2 = 4(y+1)$

$\Rightarrow y^2 - 4y - 4 = 0$

$\Rightarrow y = 2 \pm \sqrt{8}$

$\Rightarrow A\left(1-\sqrt{8},\ 2-\sqrt{8}\right)$ and $B\left(1+\sqrt{8},\ 2+\sqrt{8}\right)$

$\Rightarrow |AB| = \sqrt{32+32} = 8$

Let, PD is the length of perpendicular of $P(-1,\ 2)$ on the chord of contact AB

$\Rightarrow |PD| = \frac{|-1-1-2|}{\sqrt{2}} = 2\sqrt{2}$

Area of $\Delta PAB = \frac{1}{2} \times 2\sqrt{2} \times 8 = 8\sqrt{2},\ N = 8$

11. If $0 \le x \le 2\pi$, then the number of real values of x satisfying the equation $81^{\sin^2 x} + 81^{\cos^2 x} = 30$ is

Sol. Let $81^{\sin^2 x} = t$,

then, $81^{\cos^2 x} = 81^{1-\sin^2 x} = \frac{81}{t}$

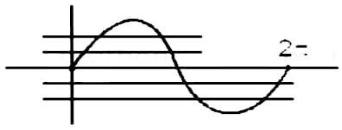
So, the given equation is $t + \frac{81}{t} = 30$

$t^2 - 30t + 81 = 0 \Rightarrow t = 3$ or 27

$81^{\sin^2 x} = 3$ or 27

$\Rightarrow 3^{4\sin^2 x} = 3^1$ or 3^3

$\Rightarrow \sin^2 x = \frac{1}{4}$ or $\frac{3}{4} \Rightarrow \sin x = \pm\frac{1}{2}, \pm\frac{\sqrt{3}}{2}$



Hence, there are 8 solutions between 0 and 2π.

12. Let the positive numbers a_1, a_2, a_3, a_4 and as be in a G.P. Let their mean and variance be 31/10 and m/n respectively, where m and n are co-prime. If the mean of their reciprocals is 31/40 and $a_3 + a_4 + a_5 = 14$, then m + n is equal to

Sol.

13. All the letters of the word "GTWENTY" are written in all possible ways with or without meaning and these words are written as in a dictionary. The serial number of the word "GTWENTY" IS

Sol. Words starting with E = 360
Words starting with GE = 60
Words starting with GN = 60
Words starting with GTE = 24
Words starting with GTN = 24
Words starting with GTT = 24
GTWENTY=1
Total = 553

SECTION 4 (Maximum Marks: 12)

- This section contains TWO (02) paragraphs.
- Based on each paragraph, there are TWO (02) questions.
- For each question, enter the correct options on OMR sheet
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 If ONLY the correct option is choosen.
  Zero Marks : 0 In all other cases.

Consider a binary string which consist only of digits 0 and 1. Let a_n be the number of binary strings of length 'n' that do not contain the sequence 11 and b_n be the number of binary strings of length 'n' that do not contain the string 111. Then (For example 0001001010 is a binary string of length 10)

14. a_n is equal to (where $n \geq 5$) (Give the numerical value of the options)
(1) $a_{n-1} + 2a_{n-2}$ (2) $a_{n-1} + a_{n-3}$ (3) $2a_{n-1} + a_{n-2}$ (4) $a_{n-1} + a_{n-2}$

Consider a binary string which consist only of digits 0 and 1. Let a_n be the number of binary strings of length 'n' that do not contain the sequence 11 and b_n be the number of binary strings of length 'n' that do not contain the string 111. Then (For example 0001001010 is a binary string of length 10)

15. b_n is equal to (where $n \geq 5$) (Give the numerical value of the options)
(1) $b_{n-1} + 2b_{n-2} + 3b_{n-3}$ (2) $b_{n-3} + b_{n-2} + b_{n-1}$ (3) $4b_{n-1} + b_{n-2} + b_{n-3}$ (4) $b_{n-1} + b_{n-2}$

Let $f(x) = x^3 + ax^2 + bx + c$ and $g(x) = x^3 + bx^2 + cx + a$, where a, b, c are integers with $c \neq 0$
Let $f(1) = 0$ and the roots of $g(x)$ are squares of the roots of $f(x)$ Then

16. $a^3 + b^2 + c =$ (Give the numerical value of the options)
(1) -1 (2) 1 (3) -3 (4) 3

Let $f(x) = x^3 + ax^2 + bx + c$ and $g(x) = x^3 + bx^2 + cx + a$, where a, b, c are integers with $c \neq 0$
Let $f(1) = 0$ and the roots of $g(x)$ are squares of the roots of $f(x)$ Then

17. $ab + bc + ca =$ (Give the numerical value of the options)
(1) -1 (2) 3 (3) 1 (4) -3