Physics Section- A

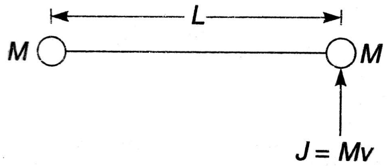
1. The following four wires are made of the same material. Which of these will have the largest extension when the same tension is applied ?

(a) Length = 50 cm, diameter = 0.5 mm
(b) Length = 100 cm, diameter = 1 mm
(c) Length = 200 cm, diameter = 2 mm
(d) Length = 300 cm, diameter = 3 mm

$$\Delta l = \frac{Fl}{AY} = \frac{Fl}{\left(\frac{\pi d^2}{4}\right)Y} \quad \text{or} \quad (\Delta l) \propto \frac{l}{d^2}$$

Now, $\frac{l}{d^2}$ is maximum in option (a)

2. Consider a body, shown in figure, consisting of two identical balls, each of mass M connected by a light rigid rod. If an impulse J = Mv is imparted to the body at one of its end, what would be its angular velocity?



(A) v/L (B) 2v/L (C) v/3L (D) v/4L

Sol. $$Mv\frac{L}{2} = \left(\frac{ML^2}{4} + \frac{ML^2}{4}\right)\omega$$

$$\omega = \frac{v}{L}$$

3. A satellite is reolving in a circular orbit at a height 'h' from the earth's surface (radius of earth R ; h << R). The minimum increase in its orbital velocity required, so that the satellite could escape from the earth's gravitational field, is close to : (Neglect the effect of atmosphere).

(A) $\sqrt{gR}\left(\sqrt{2}-1\right)$ (B) $\sqrt{2gR}$ (C) $\sqrt{gR}$ (D) $\sqrt{\frac{gR}{2}}$

Sol. as h <<< R

orbital velocity of satellite $v = \sqrt{\frac{GM_e}{(R+h)}} = \sqrt{\frac{GM_e}{R}}$

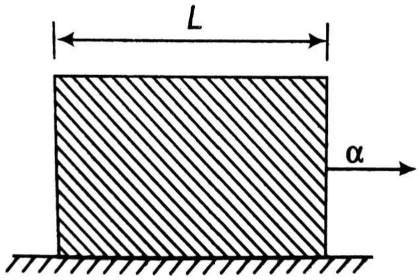
velocity required to escape earth's atmosphere :-

$$\frac{1}{2}mv_e^2 = \frac{GMm_e}{(R+h)} \Rightarrow v_e = \sqrt{\frac{2GM_e}{R}} = \sqrt{2gR} \quad \left(\frac{GM_e}{R^2} = g\right)$$

minimum increase in velocity required :- (v_e-v)

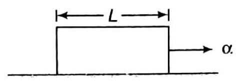
$\sqrt{gR}\left(\sqrt{2}-1\right)$

4. A uniform rod of length L and density ρ is being pulled along a smooth floor with a horizontal acceleration α . (see figure). The magnitude of the stress at the transverse cross-section through the mid-point of the rod is..........



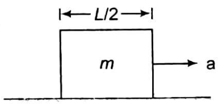
(A) $2\alpha\rho L$ **(B)** $\frac{\alpha\rho L}{2}$ (C) $\frac{\alpha\rho L}{3}$ (D) $\alpha\rho L$

Sol. Let A be the area of cross-section of the rod. FBD of rod at mid-point



Mass m = volume × density

$= \left(\frac{L}{2}.A\right)\rho$



$\therefore \quad T = m\alpha = \left(\frac{L}{2}A\rho\alpha\right)$

$\therefore$ Stress $= \frac{T}{A} = \frac{1}{2}\rho\alpha\, L$

5. A force $\vec{F} = 3x^2 y\hat{i} + x^3\hat{j}$ is acting on a mass m = 1kg kept in X-Y plane. If w be the w.d. by the force when mass moves from coordinates (0, 0) to (3, 5) under the action of the force. The value of w will be :-

(A) 135 J (B) 50 J (C) 100 J (D) 375 J

Sol. Using perfect differential method

$\vec{F} = 3x^2 y\hat{i} + x^3\hat{j} \quad \partial\vec{s} = \partial x\hat{i} + \partial y\hat{j}$

w = $\vec{F} \cdot \partial\vec{s}$

Using partial integration $\int 3x^2 y\partial x$ & $\int x^3\partial y = x^3 y$

both are equal hence w.d . = $x^3 y$

$3^3 \times 5 = 135$ J

6. A particle of mass m moving in the x-direction with speed 2v is hit by another particle of mass 2m moving in the y-direction with speed v. If the collision is perfectly inelastic, the percentage loss in the energy during the collision is close to

(a) 50% (b) 56% (c) 62% (d) 44%

Sol. Let this velocity is v_c. Then, initial momentum of system = final momentum of system

or $m(2v)\hat{\mathbf{i}} + 2m(v)\hat{\mathbf{j}} = (m+2m)\mathbf{v}_c$

$\therefore \quad \mathbf{v}_c = \frac{2}{3}(v\hat{\mathbf{i}} + v\hat{\mathbf{j}})$

$|\mathbf{v}_c|$ or v_c or speed $= \sqrt{\left(\frac{2}{3}v\right)^2 + \left(\frac{2}{3}v\right)^2}$

$= \frac{2\sqrt{2}}{3}v$

Initial kinetic energy

$$K_i = \frac{1}{2}(m)(2v)^2 + \frac{1}{2}(2m)(v)^2 = 3mv^2$$

Final kinetic energy

$$K_f = \frac{1}{2}(3m)\left(\frac{2\sqrt{2}}{3}v\right)^2 = \frac{4}{3}mv^2$$

Fractional loss $= \left(\frac{K_i - K_f}{K_i}\right) \times 100$

$$= \left[\frac{(3mv^2) - \left(\frac{4}{3}mv^2\right)}{(3mv^2)}\right] \times 100 = 56\%$$

7. A wire of length L and cross-sectional area A is made of a material of Young's modulus Y. If the wire is stretched by an amount x, the work done is

(A) $2\left[\frac{YA}{L}\right]x^2$ (B) $4\left[\frac{YA}{L}\right]x^2$ (C) $\frac{1}{2}\left[\frac{YA}{L}\right]x^2$ (D) $\left[\frac{YA}{L}\right]x^2$

Sol. $W = \frac{1}{2}Kx^2$

Here $K = \frac{YA}{L}$

$\therefore W = \frac{1}{2}\left[\frac{YA}{L}\right]x^2$

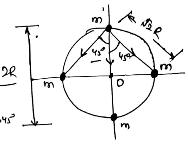
8. Four particles, each of mass M and equidistant from each other, move along a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is :

(A) $\sqrt{\frac{GM}{R}\left(1+2\sqrt{2}\right)}$ **(B)** $\frac{1}{2}\sqrt{\frac{GM}{R}\left(1+2\sqrt{2}\right)}$

(C) $\frac{1}{2}\sqrt{\frac{GM}{R}}$ (D) $\sqrt{2\sqrt{2}\frac{GM}{R}}$

Sol.

Net force towards center :-



$\Rightarrow \frac{Gm^2}{(\sqrt{2}R)^2}\cos 45° + \frac{Gm^2}{(\sqrt{2}R)^2}\cos 45°$

$+ \frac{Gm^2}{(2R)^2}$

$\Rightarrow \frac{Gm^2}{R^2}\left[\frac{4+\sqrt{2}}{4\sqrt{2}}\right] = \frac{mv^2}{R}$

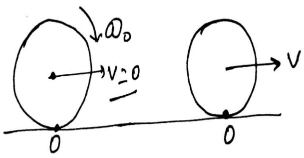
$\Rightarrow v = \frac{1}{2}\sqrt{\frac{GM}{R}\left(\frac{4+\sqrt{2}}{\sqrt{2}}\right)}$

$\Rightarrow \boxed{v = \frac{1}{2}\sqrt{\frac{GM}{R}(1+2\sqrt{2})}}$

9. A hoop of radius and mass m rotating with an angular velocity ω_0 is placed on a rough horizontal surface. The initial velocity of the centre of the hoop is zero. What will be the velocity of the centre of the hoop when it ceases to slip?

(A) $r\omega_0/4$ (B) $r\omega_0/3$ (C) $r\omega_0/2$ (D) $r\omega_0$

Sol.

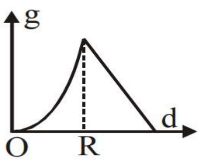


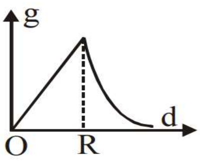
Considering conservation of angular momentum about point of contact 'O' :-

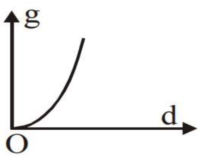
$$MR^2\omega_0 = \left(MR^2 + MR^2\right)\frac{v}{R}$$

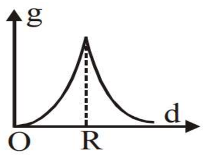
$$v = R\omega_0/2$$

10. The variation of acceleration due to gravity g with distance d from centre of the earth is best represented by (R = Earth's radius) :-

(A) 

(B) 

(C) 

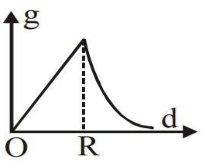
(D) 

Sol. Inside the surface of the earth $g = \frac{GM_e}{R^3}r$

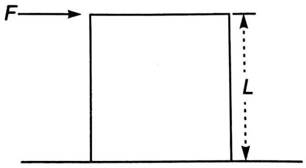
=> $g \propto r$ Linear variation

Out of the surface of the earth $g = \frac{GM_e}{r^2}$

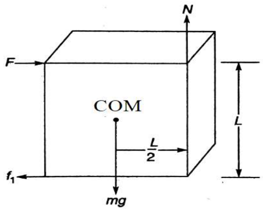
=> $g \propto \frac{1}{r^2}$



11. A cubical block of side L rests on a rough horizontal surface with coefficient of friction μ. A horizontal force F is applied on the block as shown. If the coefficient of friction is sufficiently high, so that the block does not slide before toppling, the minimum force required to topple the block is



(A) infinitesimal (B) mg/4 (C) mg/2 (D) mg(1-μ)

Sol. 

$$FL > \frac{MgL}{2} \Rightarrow F > \frac{Mg}{2} \text{ hence } F_{min} = \frac{Mg}{2}$$

12. A satellite is moving with a constant speed V in a circular orbit about the earth. An object of mass 'm' is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of its ejection, the kinetic energy of the object is :-

(A) $\frac{1}{2}mV^2$ (B) mV^2 (C) $\frac{3}{2}mV^2$ (D) $2mV^2$

Sol. Total energy $= \frac{-GM_e m}{x} + K.E$.

(where x is the hight from the centre of the earth)

while total energy at infinity is zero

$\frac{-GM_e m}{x} + K.E. = 0$ (orbital velocity of satellite is $V = \sqrt{\frac{GM_e}{x}}$)

K.E. = mV^2

13. A rod of weight W is supported by two parallel knife edges A and B and is in equilibrium in a horizontal position. The knives are at a distance d from each other. The centre of mass of the rod is at distance x from A. The normal reaction on B is

(A) $W\left(\frac{d-x}{d}\right)$ (B) W (C) $\frac{Wx}{d}$ (D) $\frac{W}{d}$

Sol. $N_A + N_B = W$ & $N_A x = N_B (d - x) \Rightarrow N_B = \frac{Wx}{d}$

14. A smooth sphere A is moving on a frictionless horizontal plane with angular velocity ω and centre of mass velocity v. It collides elastically and head on with an identical sphere B at rest. Neglect friction everywhere. After the collision their angular speeds are ω_A and ω_B respectively. Then,

(A) $\omega_A < \omega_B$ **(B)** $\omega_B = 0$ (C) $\omega_B = \omega_A$ (D) $\omega_B = \omega$

Sol. Since, it is head on elastic collision between two identical spheres, they will exchange their linear velocities i.e., A comes to rest and B starts moving with linear velocity v. As there is no friction any where, torque on both the spheres about their centre of mass is zero and their angular velocities remain unchanged. Therefore, $\omega_B = 0$

15. The mass of a spaceship is 1000 kg. It is to be launched from the earth's surface out into free space. The value of 'g' and 'R' (radius of earth) are 10 m/s^2 and 6400 km respectively. The required energy for this work will be :-

(A) 6.4×10^{10} Joules (B) 6.4×10^{11} Joules
(C) 6.4×10^{8} Joules (D) 6.4×10^{9} Joules

Sol. Using energy conservation

$$\frac{-GM_e m}{R} + K.E. = 0$$

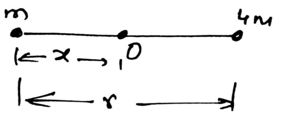
K.E. = mgR $\quad \left(g = \frac{GM_e}{R^2} \right)$

$= 6.4 \times 10^{10}$ Joules

16. Two bodies of masses m and 4m are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is :-

(A) $-\frac{6Gm}{r}$ (B) $-\frac{9Gm}{r}$ (C) zero (D) $-\frac{4Gm}{r}$

Sol.



At point 'O' $\frac{Gm}{x^2} = \frac{G4m}{(r-x)^2}$

=> x = r/3

Total gravitational potential at point 'O' : $-\frac{3Gm}{r} - \frac{12Gm}{2r}$

=> $V_0 = -\frac{9Gm}{r}$

17. The height at which the acceleration due to gravity becomes $\frac{g}{9}$ (where g = the acceleration due to gravity on the surface of the earth) in terms of R, the radius of the earth, is :-

(A) $\frac{R}{2}$ (B) $\sqrt{2}R$ (C) $2R$ (D) $\frac{R}{\sqrt{2}}$

Sol. $\frac{g}{9} = g\left(\frac{R}{R+h}\right)^2$

$h = 2R$

18. A satellite S is moving in an elliptical orbit around the earth. The mass of the satellite is very small compared to the mass of the earth

(a) the acceleration of S is always directed towards the centre of the earth
(b) the angular momentum of S about the centre of the earth changes in direction, but its magnitude remains constant
(c) the total mechanical energy of S varies periodically with time
(d) the linear momentum of S remains constant in magnitude

Sol. Force on satellite is always towards earth, therefore, acceleration of satellite S is always directed towards centre of the earth. Net torque of this gravitational force F about centre of earth is zero. Therefore, angular momentum (both in magnitude and direction) of S about centre of earth is constant throughout. Since, the force F is conservative in nature, therefore mechanical energy of satellite remains constant. Speed of S is maximum when it is nearest to earth and minimum when it is farthest.

19. Imagine a light planet revolving around a very massive star in a circular orbit of radius R with a period of revolution T. If the gravitational force of attraction between the planet and the star is proportional to $R^{-5/2}$, then

(A) T^2 is proportional to R^2
(B) T^2 is proportional to $R^{7/2}$
(C) T^2 is proportional to $R^{3/2}$
(D) T^2 is proportional to $R^{3.75}$

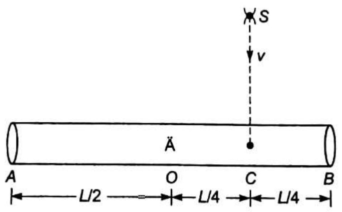
Sol. $\frac{mv^2}{R} \propto R^{-5/2}$

$\therefore \quad v \propto R^{-3/4}$

Now, $T = \frac{2\pi R}{v}$ or $T^2 \propto \left(\frac{R}{v}\right)^2$

or $T^2 \propto \left(\frac{R}{R^{-3/4}}\right)^2$ or $T^2 \propto R^{7/2}$

20. A homogeneous rod AB of length L and mass M is pivoted at the centre O in such a way that it can rotate freely in the vertical plane (figure). The rod is initially in the horizontal position. An insect S of the same mass M falls vertically with speed v on the point C, midway between the points O and B. Immediately after falling, the insect moves towards the end B such that the rod rotates with a constant angular velocity ω. Angular velocity ω in terms of v and L just after the fall of the insect will be



(A) $\frac{8v}{7L}$ (B) $\frac{12v}{9L}$ (C) $\frac{12v}{7L}$ (D) $\frac{8v}{15L}$

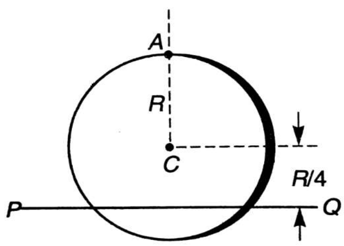
Sol. Applying angular momentum conservation just before and after fall of the insect

$$\frac{MvL}{4} = \left(\frac{ML^2}{16} + \frac{ML^2}{12}\right)\omega$$

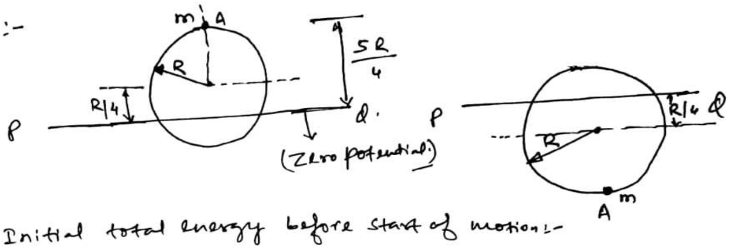
$$\Rightarrow \omega = \frac{12v}{7L}$$

Physics Section - B

1. A uniform circular disc has radius R = 2 m and mass m = 2kg . A particle, also of mass m, is fixed at a point. A on the edge of the disc as shown in the figure. The disc can rotate freely about a horizontal chord PQ that is at a distance R/4 from the centre C of the disc. The line AC is perpendicular to PQ. Initially the disc is held vertical with the point A at its highest position. It is then allowed to fall, so that it starts rotation about PQ. If v be the linear speed of the particle as it reaches its lowest position. v =_____m/s (Take g = 10m/s^2)



Sol^n :-



Initial total energy before start of motion:-

$$\Delta E_i = mg\frac{R}{4} + mg\frac{5R}{4} = \frac{6mgR}{4}$$

when system reaches the lowest position:-

$$\Delta E_f = \frac{1}{2} I_{total}\,\omega^2 - \frac{6mgR}{4}$$

$$I_{total} = I_{Disc/diameter} + m\left(\frac{R}{4}\right)^2 + m\left(\frac{5R}{4}\right)^2$$

$$I_{total} = \frac{mR^2}{4} + \frac{mR^2}{16} + \frac{25mR^2}{16} \Rightarrow \frac{30mR^2}{16}$$

using energy conservation:-

$$\Delta E_i = \Delta E_f$$

$$\Rightarrow \quad 3mgR = \frac{15mR^2}{16}\omega^2 \Rightarrow \boxed{\omega = \sqrt{\frac{16g}{5R}}}$$

Linear velocity of particle when it reaches lowest position $\quad v = \omega \times \frac{5R}{4} \Rightarrow \boxed{v = \sqrt{5gR}}$

$$\boxed{v = \sqrt{5\times10\times2} \Rightarrow 10\,m/sec}$$

2. Two rods of different materials having coefficients of thermal expansion α_1, α_2 and Young's moduli Y_1, Y_2 respectively are fixed between two rigid massive walls. The rods are heated such that they undergo the same increase in temperature. There is no bending of the rods. If $\alpha_1 : \alpha_2 = 2:3$ the thermal stresses developed in the two rods are equal provided. Calculate $4Y_1$ =______ Y_2.

Sol. Thermal stress $= Y\alpha\Delta\theta$

$$\sigma_1 = Y_1\alpha_1\Delta\theta_1 \ \& \ \sigma_2 = Y_2\alpha_2\Delta\theta_2$$

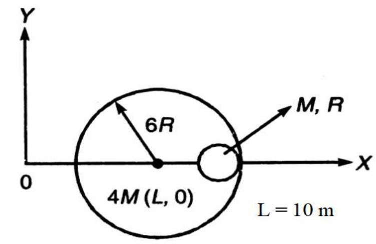
given, $\sigma_1 = \sigma_2$ & $\Delta\theta_1 = \Delta\theta_2$

$$Y_1\alpha_1\Delta\theta_1 = Y_2\alpha_2\Delta\theta_2 \Rightarrow Y_1 = \frac{\alpha_2}{\alpha_1}Y_2$$

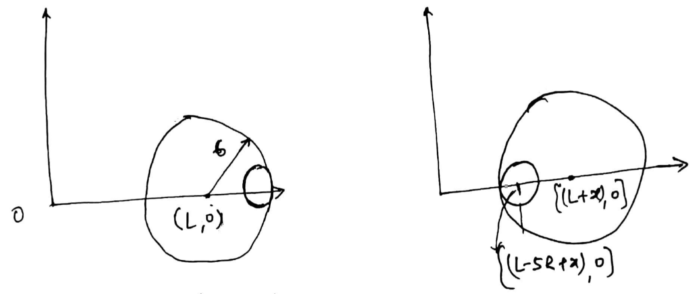
$$\Rightarrow 2Y_1 = 3Y_2$$

$\therefore\ 4Y_1 = 6Y_2$

3. A small sphere of radius R = 1 m is held against the inner surface of a larger sphere of radius 6R. The masses of large and small spheres are 4M and M(where M = 1kg) respectively. This arrangement is placed on a horizontal table. There is no friction between any surfaces of contact. The small sphere is now released, the x-coordinates of the centre of the larger sphere when the smaller sphere reaches the other extreme position will be _____m.



Sol^n



$$\text{COM} = \frac{4\times L + 1(L+5R)}{5} \Rightarrow \frac{5L+5R}{5} \quad \text{— eq}^n\ (1)$$

When smaller mass reaches the left corner

$$\text{COM} = \frac{4(L+x) + 1(L-5R+x)}{5} \Rightarrow \frac{5L+5x-5R}{5} \quad \text{— eq}^n\ (2)$$

using eq^n (1) & (2)

$$\frac{\cancel{5}L+5R}{\cancel{5}} = \frac{\cancel{5}L+5x-5R}{\cancel{5}} \Rightarrow \boxed{x = 2R}$$

final x-coordinate :- $L+2R \Rightarrow 12\text{ m}$.

4. One end of a horizontal thick copper wire of length 3L and radius 3R is welded to an end of another horizontal thin copper wire of length L and radius R. When the arrangement is stretched by applying forces at two ends, the ratio of the elongation in the thin wire to that in the thick wire is______.

Sol. $\Delta l = \frac{FL}{AY} \Rightarrow \frac{FL}{\pi r^2 Y} \Rightarrow \Delta l \propto \frac{L}{r^2}$

$$\frac{\Delta l_1}{\Delta l_2} = \frac{L/r^2}{3L/(3r)^2}$$

$= 3$

5. If V_0 be the minimum velocity required to project a mass m = 1000kg from the surface of earth to infinity. Calculate $\sqrt{2}V_0$ =______km/s. (Take R_e = 6400km)

Sol. Using energy conservation

$$-\frac{GM_e m}{R} + \frac{1}{2}mV_0^2 = 0$$

$$\frac{GM_e m}{R} = \frac{1}{2}mV_0^2 \Rightarrow V_0 = \sqrt{2gR} \text{ since } \left\{g = \frac{GM_e}{R^2}\right\}$$

$V_0 = 8\sqrt{2} km/\sec$

= 16 km/sec

6. If T_1 be the number of days in a year when the distance between the sun and the earth has been doubled. Calculate $\sqrt{2}T_1$ = ______days

Sol. $T^2 \propto d^3$

$\Rightarrow T^2 = Kd^3 \;\&\; T_1^2 = K(2d)^3$

$\Rightarrow T_1 = 2\sqrt{2}T$

$\sqrt{2}T_1$ = 1460 days

7. A satellite of mass m = 800 kg revolves around the earth of radius R = 6400km at a height x = 29600 km from its surface. If g is the acceleration due to gravity on the surface of the earth. If V_0 be the orbital speed of the satellite then calculate $\frac{6V_0}{10\sqrt{10}}$ =________m/s.

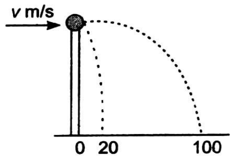
Sol. $$\frac{GM_e m}{(R+x)^2} = \frac{mV_0^2}{R+x}$$

$$V_0 = \sqrt{\frac{gR^2}{R+x}} \text{ since } \left\{g = \frac{GM_e}{R^2}\right\}$$

$$V_0 = \frac{6400\sqrt{10}}{6} \text{ m/s}$$

$$\frac{6V_0}{10\sqrt{10}} = 640 \text{ m/s}$$

8. A ball of mass 0.2 kg rests on a vertical post of height 5 m. A bullet of mass 0.01 kg, travelling with a velocity v m/s in a horizontal direction, hits the centre of the ball. After the collision, the ball and bullet travel independently. The ball hits the ground at a distance of 20 m and the bullet at a distance of 100 m from the foot of the post. The initial velocity v of the bullet is____m/s



Sol. Let V_B & V_1 be the speeds of bullet and ball after the collision and t_B and t_1 be the time taken by them to hit the ground.

Using eq^n $-H = -\frac{1}{2}gt^2$ => $t_B = t_1 = \sqrt{\frac{2H}{g}}$

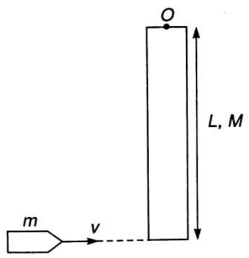
$$V_B = 100 \times \sqrt{\frac{g}{2H}} \text{ and } V_1 = 20 \times \sqrt{\frac{g}{2H}}$$

using momentum conservation

$$m_B \times V = m_B \times V_B + M \times V_1$$

$$V = V_B + \frac{M}{m_B} \times V_1 = 500 \text{ m/s}$$

9. A rod of length L = 1m and mass M = 10 kg is hinged at point O. A small bullet of mass m = 1kg hits the rod with velocity $\vec{v} = 26\hat{i}$ m/s shown in the figure. The bullet gets embedded in the rod. If ω be the angular velocity of the system just after impact. ω = _______rad/s



Sol. Using conservation of angular momentum just before & after the impact about hinged point 'O'

$$mVL = (I_{rod/o} + mL^2)\,\omega$$

=> $$mvL = \left(\frac{ML^2}{3} + mL^2\right)\omega \Rightarrow \omega = \frac{3mv}{(M+3m)L}$$

ω = 6 rad/sec

10. A particle is projected vertically upwards from the surface of earth (radius R = 6400km)with a kinetic energy equal to half of the minimum value needed for it to escape. If h be the maximum height to which it rises above the surface of earth . Calculate 2h =_________km.

Sol. K.E. needed toescape the earth's surface $\frac{GM_e m}{R}$

while energy impacted to the particle :- $\frac{GM_e m}{2R}$

using conservation of energy :-

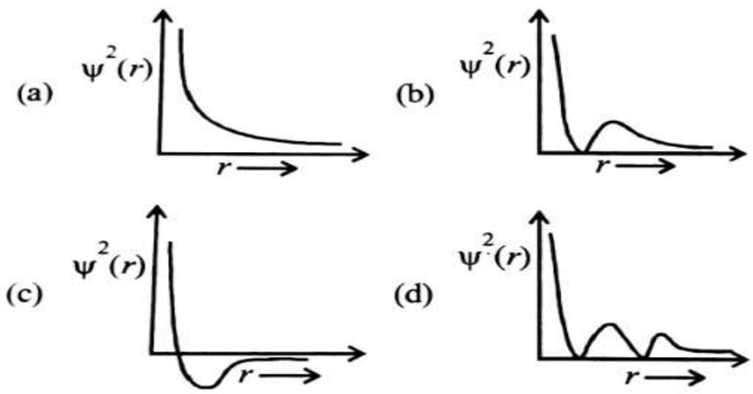
$$-\frac{GM_e m}{R} + \frac{GM_e m}{2R} = -\frac{GM_e m}{(R+h)}$$

=> $$\frac{1}{2R} = \frac{1}{R+h} \Rightarrow h = R = 6400km$$

2h = 12800km

Chemistry Section -A

1. Which of the following is the correct plot for the probability density ψ^2 (r) as a function of distance 'r' of the electron form the nucleus for 2s orbital?



ans - B

2. When $NaNO_3$ is heated in a closed vessel, O_2 is liberated and $NaNO_2$ is left behind. At equilibrium
(a) addition of $NaNO_2$ favours reverse reaction
(b) addition of $NaNO_3$ favours forward reaction
(c) increasing temperature favours forward reaction
(d) increasing pressure favours forward reaction
Sol. increasing temperature favours forward reaction due to endothermic reaction

3. When equal volumes of the following solutions are mixed, precipitation of AgCl (K_{sp} 1.8×10^{-10}) will only with
(A) 10^{-4} M (Ag^+) and 10^{-4} M (Cl^-) (B) 10^{-5} M (Ag^+) and 10^{-5} M (Cl^-)
(C) 10^{-6} M (Ag^+) and 10^{-6} M (Cl^-) (D) 10^{-10} M (Ag^+) and 10^{-10} M (Cl^-)

Sol. When equal volumes of two solutions are mixed that time solution concentration is decreased by 2 unit.
NOTE: The condition for precipitation of sparingly soluble salt is $\Rightarrow K_{I.P.} > K_{S.P}$
· Ionic product > solubility product.
· $K_{SP}(AgCl) = 1.8 \times 10^{-11}$. so, $I.P > 1.8 \times 10^{-11}$ for precipitation
(A) $[Ag^+] = \frac{10^{-4}}{2}$, $[Cl^-] = \frac{10^{-4}}{2}$.
Ionic product $= \underbrace{\frac{10^{-4}}{2} \times \frac{10^{-4}}{2}}_{[Ag^+] \times [Cl^-]} = 0.25 \times 10^{-8}$
so, only first option (A) having I.P value is greater than solubility product.
· Option (A) is correct.

4. The orbital angular momentum for an electron revolving in an orbit is given by $\sqrt{l(l+1)}.\frac{h}{2\pi}$. This momentum for an s-electron will be given by

(A) $\frac{h}{2\pi}$ (B) Zero (C) $\sqrt{2}.\frac{h}{2\pi}$ (D) $+\frac{1}{2}.\frac{h}{2\pi}$

Sol. For s-electron, azimuthal quantum number(l) is zero(0). so, $l = 0$.

• Orbital angular momentum is $= \sqrt{l(l+1)} \cdot \frac{h}{2\pi}$

$= 0$.

• option (B) is correct.

5. If pK_b for fluoride ion at 25ºC is 10.83, the ionisation constant of hydrofluoric acid in water at this temperature is

(a) 1.74×10^{-5} (b) 3.52×10^{-3} **(c)** 6.75×10^{-4} (d 5.38×10^{-1}

Sol. For weak Acid

$\boxed{Ka = \frac{Kw}{Kb}}$

$pKa + pKb = pKw$

$pKa = pKw - pKb$

$pKa = 14 - 10.83$ (given)

$pKa = 3.17$

$-\log Ka = 3.17$

$Ka = \text{Antilog } 3.17$ (Antilog $= 6.75 \times 10^{-4}$)

$Ka = 6.75 \times 10^{-4}$

6. $2Al(s) + 6HCl(aq) \rightarrow 2Al^{3+}(aq) + 6Cl^{-}(aq) + 3H_2(g)$

(a) 11.2 L $H_2(g)$ at STP is produced for every mole of HCl(aq) consumed

(b) 6 L HCl(aq) is consumed for every 3 L of $H_2(g)$ produced

(c) 33.6 L $H_2(g)$ is produced regardless of temperature and pressure for every mole of Al that reacts

(d) $67.2H_2(g)$ at STP is produced for every mole of Al that reacts

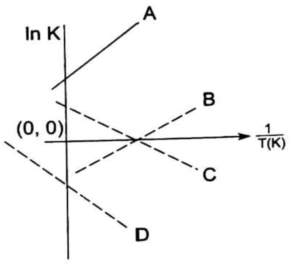
Sol. $2Al(s) + 6HCl(aq) \longrightarrow 2Al^{3+}(aq) + 6Cl^{-}(aq) + 3H_2$

6 mole 3 mole

1 mole $\frac{3}{6} = \frac{1}{2}$ mole

$\therefore \frac{1}{2}$ mole of H_2 gives 11.2 L H_2

7. Which of the following lines correctly show the temperature dependence of equilibrium constant K, for an exothermic reaction?



(a) A and D (b) A and B (c) B and C (d) C and D

Sol.
$\ln K = \frac{-\Delta H^\circ}{RT} + \frac{\Delta S^\circ}{R}$

$y = -m x + c$

$\boxed{y = -\Delta H \frac{1}{T} + c}$

A/c to question.
$\Delta H = -ve$; Exothermic.
slope is +ve
Ans → (B) (A & B)

8. For the following reactions, equilibrium constants are given:
$S(s) + O_2(g) \rightleftharpoons SO_2(g); K_1 = 10^{52}$
$2S(s) + 3O_2(g) \rightleftharpoons 2SO_3(g); K_2 = 10^{129}$
The equilibrium constant for the reaction,
$2SO_{2(g)} + O_{2(g)} \rightleftharpoons 2SO_{3(g)}, K_3 = ?$

(A) 10^{154} (B) 10^{181} **(C)** 10^{25} (D) 10^{77}
Sol.

$S(s) + O_2(g) \rightleftharpoons SO_2(g); K_1 = 10^{52}$
$2S(s) + 3O_2(g) \rightleftharpoons 2SO_3(g); K_2 = 10^{129}$
$\therefore 2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g); K_3 = ??$

$\therefore K_3 = \frac{K_2}{K_1^2} = \frac{10^{129}}{(10^{52})^2} = 10^{25}$ (Ans)

option (C) is correct.

9. At 90°C, pure water has $[H_3O^+] = 10^{-6}$ mole/litre. The value of K_w at 90°C is

(A) 10^{-6} (B) 10^{-8} **(C)** 10^{-12} (D) 10^{-14}
Sol.

10. An example of a reversible reaction is
(a) $Pb(NO_3)_2$ (aq) + 2NaI (aq) = PbI_2 (s) + $2NaNO_3$ (aq)
(b) $AgNO_3$ (aq) + HCl (aq) = AgCl (s) + HNO_3 (aq)
(c) 2Na (s) + H_2O (1) = 2NaOH (aq) + H_2 (g)
(d) KNO_3 (aq) + NaOH (aq) = KOH (aq) + $NaNO_3$ (aq)

11. The initial rate of hydrolysis of methyl acetate (1 M) by a weak acid (HA,1 M is $\left(\frac{1}{100}\right)$th of that of a strong acid (HX, 1 M) at 25ºC. The K_a(HA) value is

(A) 1×10^{-4} (B) 1×10^{-5} (C) 1×10^{-6} (D) 1×10^{-3}

Sol. $HA \rightleftharpoons H^+ + A^-$

$K_a = \frac{\alpha^2 c}{1-\alpha} = (10^{-2})^2 \times 1$

$K_a = 10^{-4} \times 1$

$\boxed{K_a = 1 \times 10^{-4}}$

12. Which of the following are Lewis acids?

(a) BCl_3 and $AlCl_3$ (b) PH_3 and $BC1_3$

(c) AICI, and $SiCl_4$ (d) PH_3 and $SiCl_4$

Sol. ans:- a

13. The K_{sp} for the following dissociation is 1.6×10^{-5}

$PbCl2(s) \rightleftharpoons Pb^{2+}(aq) + 2Cl^-(aq)$

Which of the following choices is correct for a mixture of 300mL 0.134 M $Pb(NO_3)_2$ and 100 mL 0.4 M NaCl?

(A) Not enough data provided (B) $Q < K_{sp}$ **(C)** $Q > K_{sp}$ (D) $Q = K_{sp}$

Sol. $PbCl_2(s) \rightleftharpoons Pb^{2+}(aq) + 2Cl^-(aq)$

$K_{sp} = [Pb^{2+}]\cdot[Cl^-]^2 = 1.6 \times 10^{-5}$

In questions, 300 ml of 0.134M $Pb(NO_3)_2$ mixed with 100 ml 0.4M NaCl. After mixing,

$[Pb^{2+}] = \frac{300 \times 0.134}{400} = 0.1005M$.

$[Cl^-] = \frac{100 \times 0.4}{400} = 0.1M$.

Here, ionic product (I.P.) is denoted by Q.

$Q = [Pb^{2+}]\cdot[Cl^-]^2 = 0.1005 \times (0.1)^2$

$= 0.001005$.

$= 1.005 \times 10^{-3}$.

$\therefore Q > K_{sp}$

• Option (C) is Correct.

14. Consider the following reduction processes:

$Zn^{2+} + 2e^- \rightarrow Zn(s)$; $E^o = -0.76V$

$Ca^{2+} + 2e^- \rightarrow Ca(s)$; $E^o = -2.87$ V

$Mg^{2+} + 2e^- \rightarrow Mg(s)$; $E^o = -2.36$ V

$Ni^{2+} + 2e^- \rightarrow Ni(s)$; $E^o = -0.25$ V

The reducing power of the metals increases in the order:

(a) Ca < Zn < Mg < Ni
(b) Ni < Zn < Mg < Ca
(c) Zn < Mg < Ni < Ca
(d) Ca < Mg < Zn < Ni

Sol.

$Zn^{2+} + 2e \rightarrow Zn(s)$; $E^o = -0.76V$

$Ca^{2+} + 2e \rightarrow Ca(s)$; $E^o = -2.87V$

$Mg^{2+} + 2e \rightarrow Mg(s)$; $E^o = -2.36V$

$Ni^{2+} + 2e \rightarrow Ni(s)$; $E^o = -0.25V$

Here, All E^o are standard reduction potential.
As we know that lower the standard reduction potential having higher reducing power.

$\therefore E^o \propto \dfrac{1}{\text{reducing power}}$

standard reduction potential.

$\because$ Reducing power $\Rightarrow$ Ca > Mg > Zn > Ni.

• Option (b) is correct.

15. Which of the following statement(s) is/are correct?

(A) The pH of 1×10^{-8} M HCl solution is 8.

(B) The conjugate base $H_2PO_4^-$ is HPO_4^{2-}

(C) K_w increases with increase in temperature.

(D) When a solution of weak monoprotic acid is titrated against a strong base at half neutralisation point, $pH = \frac{1}{2} pK_a$

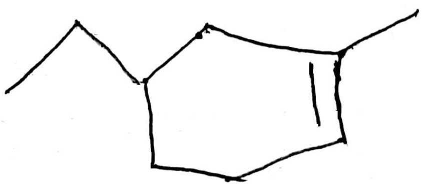
Choose the correct answer from the option given below.

(a) (B), (C), (D)
(b) (A),(D)
(c) (A), (B), (C)
(d) (B),(C)

16. Copper becomes green when exposed to moist air for a long period. This is due to:

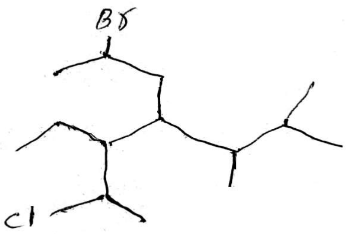
(a) the formation of a layer of cupric oxide on the surface of copper.
(b) the formation of a layer of basic carbonate of copper on the surface of copper.
(c) the formation of a layer of cupric hydroxide on the surface of copper.
(d) the formation of basic copper sulphate layer on the surface of the metal.

17. Correct the I.U.P.A.C name of the compound



(A) 1-methyl - 3-ethylcyclohexene
(B) 5- ethyl - 1-methylcyclohexene
(C) 2- ethyl - 4- methylcyclohexene
(D) 3- ethyl - 1 - methylcyclohexene

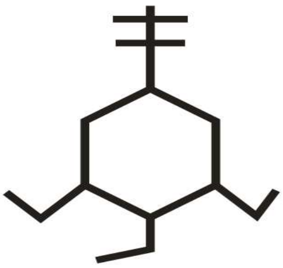
18. How many total number of substituent are present in the compound



(A) 3 **(B)** 4 (C) 5 (D) 6

ans: - 4

19. Total number of secondary and quaternary Carbon & Hydrogen of the compound ?



	secondary Carbon	quaternary Carbon	secondary Hydrogen	quaternary Hydrogen
(A)	5	2	10	0
(B)	5	3	10	0
(C)	4	4	10	4
(D)	5	4	4	10

Sol. $4^\circ C = 2$, $4^\circ H = 0$
$2C = 5$ $2^\circ H = 10$

20. Select the decreasing order of priority of the functional group
[Based on I.U.P.A.C]

(A) $-CH=O > -\underset{\overset{\|}{O}}{C}- > OH > -\underset{\overset{\|}{O}}{C}-OH$

(B) $-\underset{\overset{\|}{O}}{C}-OH > -C\equiv N > -O- > -OH$

(C) $-\underset{\overset{\|}{O}}{C}-OH > -\underset{\overset{\|}{O}}{C}-O- > -\underset{\overset{\|}{O}}{C}-O-\underset{\overset{\|}{O}}{C}- > -C\equiv N$

(D) $-\underset{\overset{\|}{O}}{C}-OH > -\underset{\overset{\|}{O}}{C}-O-\underset{\overset{\|}{O}}{C}- > \underset{\overset{\|}{O}}{C}-O-R > -\underset{\overset{\|}{O}}{C}-X$

Chemistry Section -B

1. Consider the following reaction:

$$xMnO_4^- + yC_2O_4^{2-} + zH^+ \rightarrow xMn^{2+} + 2yCO_2 + \frac{z}{2}H_2O$$

The value of $x + y + z =$ _______.

Sol.

$$\overset{+7}{Mn}O_4^- + \overset{+3}{C_2}O_4^{2-} + H^+ \longrightarrow Mn^{2+} + \overset{+4}{CO_2} + H_2O$$

CON = 2

CON = 5

$$2MnO_4^- + 5C_2O_4^{2-} + 16H^+ \longrightarrow 2Mn^{2+} + 5CO_2 + 8H_2O$$

$$2 + 5 + 16 = 23$$

Ans - 23

2. Consider the ground state of Cr atom (X = 24). The sum of number of electrons with Azimuthal Quantum numbers, l = 1 and 2 is _______.

Sol.

$${}_{24}Cr = 1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,3d^5\,4s^1$$

$l = 1$ & 2

$l = p$ & d

$\therefore$ $2p^6 + 3p^6 + 3d^5$

Total electron = $6 + 6 + 5 = 17$

ans;- 17

3. The solubility of A_2X_3 is y mol/dm^3. If My^n be the solubility product of the molecule. Then M + n = ____.

Sol.

$$A_2X_3 \rightleftharpoons 2A^{+3} + 3X^{-2}$$

$$(1-s) \qquad [2s]^2\ [3s]^3$$

$$K_{sp} = 4s^2 \times 27s^3$$

$$K_{sp} = (108)s^5$$

$= 108\ s^5$

or $108\ y^5$

or $\underbrace{M \, y^n} = M + n$

or $108 + 5 = 113$

ans :- 113

4. Consider the following complex compounds

$[Cr(H_2O)_6]Cl_3$, $[Cr(C_6H_6)_2]$ and $K_2[Cr(CN)_2O_2(O)_2(NH_3)]$

The sum of oxidation states of Cr is ______.

Sol. $[Cr(H_2O)_6Cl_3]$, $[Cr(C_6H_6)_2]$, $K_2[Cr(CN)_2(O_2)_2O_2(NH_3)]$

$+3 + 0 + 6$

ans 9

5. The number of orbitals associated with Quantum numbers n = 5, $m_s = +\frac{1}{2}$ is _____.

Sol. Number of Orbitals = n^2 (n = 5, given)

$= (5)^2 = 25$

ans :- 25

6. The five successive ionization enthalpies of an element are 800, 2427, 3658, 25024 and 32824 kJ mol^{-1}. The number of valence electrons in the element is_____.

Sol. IE_3 and IE_4 is maximum value

there is drastic change in the I.E.

From 3rd to 4th

So elements containing 3

Valence electron

after losing 3 electron it acquires

stable inert gas configuration

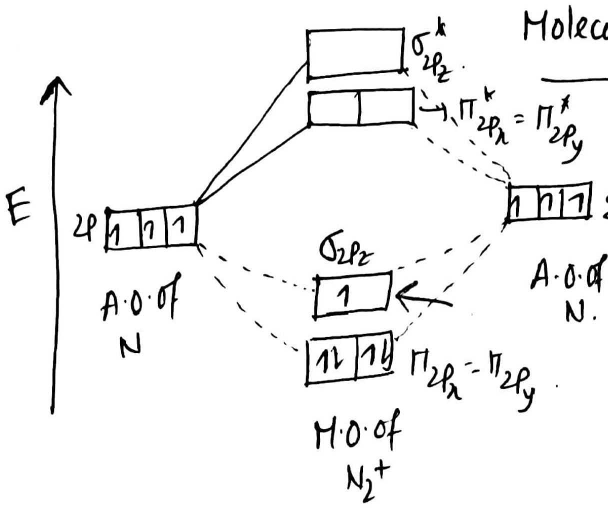
ans :- 3

7. In the molecular orbital diagram for the molecular ion , N_2^+, the number of electrons in the σ_{2p} molecular orbital is_____.

Sol.

Molecular axis z-axis

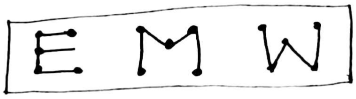
$N: 1s^2 2s^2 2p^3$



so, the number of electrons in the σ_{2p_z} molecular orbitals is ⇒ One (1)

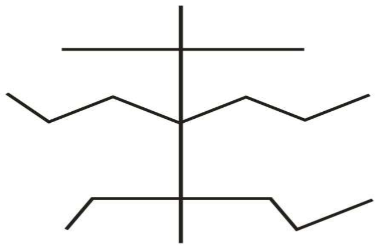
8. How many of the following alphabat have a five membered chain as longest chain ?
AEITMKFHVWXYZ

Sol. only three (3) alphabat have a five membered chain as longest chain.

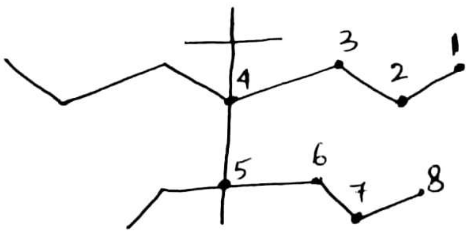


ans:- 3

9. Total number of carbon atoms present in parent or main chain is



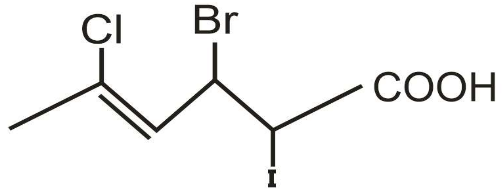
Sol.



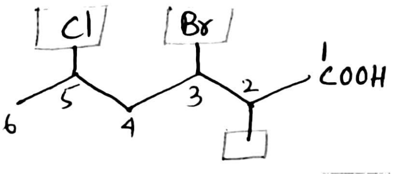
so, total number of carbon atoms present in main chain is ⇒ 8

ans :- 8

10. Total number of substituent present in the compound



Sol.



$\Rightarrow$ Total number of substituent present in the comp is $\underline{3}$

Mathematics Section - A

1. Le a, ß be the roots of $x^2 - x + p = 0$ and γ, δ the roots of $x^2 - 4x + q = 0$. If $\alpha, \beta, \gamma, \delta$ are in G.P., then the integral values of p and q respectively, are

(A) – 2, – 32 (B) – 2,3 (C) – 6, 3 (D) – 6, – 32

Sol.
$\alpha = a$; $\beta = ar$; $\gamma = ar^2$; $S = ar^3$
$\alpha + \beta = a(1 + r) = 1$; $\alpha\beta = a^2r = p$
$\gamma + \delta = ar^2(1 + r) = 4$; $\gamma\delta = a^2r^5 = q$
$\Rightarrow$ $r^2 = 4$
$\Rightarrow$ $\gamma = 2$or $r = -2$
$\Rightarrow$ $a = -1$ roots are $-1, 2, -4, 8$
$\Rightarrow$ $p = -2$; $q = -32$

2. Let V_r denote the sum of first r terms of an arithmetic progression (A.P.) whose first term is r and the common difference is (2r - 1). The sum $V_1 + V_2 + \ldots\ldots + V_n$ is

(A) $\frac{1}{12}n(n+1)(3n^2 - n + 1)$ (B) $\frac{1}{12}n(n+1)(3n^2 + n + 2)$

(C) $\frac{1}{2}n(2n^2 - n + 1)$ (D) $\frac{1}{3}(2n^3 - 2n + 3)$

Sol.

$$V_r = \frac{r}{2}\left[2r + (r-1)(2r-1)\right]$$

$$= \frac{r}{2}\left[2r^2 - r + 1\right] = r^3 - \frac{1}{2}r^2 + \frac{1}{2}r$$

$$V_1 + V_2 + \cdots V_n = \sum_{r=1}^{n} V_r = \sum_{r=1}^{n}\left(r^3 - \frac{r^2}{2} + \frac{r}{2}\right)$$

$$= \left[\frac{n(n+1)}{2}\right]^2 - \frac{1}{2}\,\frac{n(n+1)(2n+1)}{6} + \frac{1}{2}\,\frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{2}\left[\frac{n^2+n}{2} - \frac{2n+1}{6} + \frac{1}{2}\right]$$

$$= \frac{1}{12}\,n(n+1)\left\{3n^2 + 3n - 2n - 1 + 3\right\}$$

$$= \frac{1}{12}\,n(n+1)(3n^2 + n + 2)$$

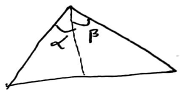
3. The vertices angle of a triangle is divided into two parts, such that the tangent of one part is 3 times the tangent of the other and the difference of these parts is 30°, then the triangle is

(A) isosceles (B) right angled (C) obtuse angled (D) None of these

Sol.

$\tan\alpha = 3\tan\beta$ ...(i)

$\tan(\alpha-\beta) = \frac{1}{\sqrt{3}}$ ...(ii)



$\frac{3\tan\beta - \tan\beta}{1+3\tan^2\beta} = \frac{1}{\sqrt{3}}$

$\Rightarrow 2\sqrt{3}\tan\beta = 1+3\tan^2\beta$

$\Rightarrow (\sqrt{3}\tan\beta - 1)^2 = 0$

$\Rightarrow \tan\beta = 1/\sqrt{3}$

$\Rightarrow \tan\alpha = \sqrt{3}$

$\Rightarrow \alpha+\beta = 90°$

$\Rightarrow$ right angled.

4. The sides a, b, c of a triangle ABC are the roots of $x^3-11x^2+38x-40=0$ then $\sum \frac{\cos A}{a} =$

(A) $\frac{3}{4}$ (B) 1 (C) $\frac{9}{16}$ (D) None of these

Sol.

$a+b+c = 11$

$ab+bc+ca = 38$

$abc = 40$

$\sum \frac{\cos A}{a} = \frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c}$

$= \frac{b^2+c^2-a^2}{2abc} + \frac{a^2+c^2-b^2}{2abc} + \frac{a^2+b^2-c^2}{2abc}$

$= \frac{(a+b+c)^2 - 2(ab+bc+ca)}{2abc} = \frac{(11)^2-2(38)}{80}$

$= \frac{45}{80} = 9/16$

5. If $\cos A = \frac{\sin B}{2\sin C}$, then ΔABC is

(A) equilateral (B) isosceles (C) right angled (D) None of these

Sol.

$\cos A = \frac{\sin B}{2\sin C} \Rightarrow \frac{b^2+c^2-a^2}{\cancel{2}b\cancel{c}} = \frac{\cancel{c}}{\cancel{2}\cancel{b}}$

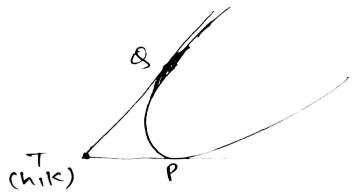
$\Rightarrow b^2+c^2-a^2 = c^2 \Rightarrow b^2 = a^2 \Rightarrow a = b$

$\Rightarrow \Delta$ is isosceles

6. TP & TQ are tangents to the parabola, $y^2 = 4ax$ at P & Q. If the chord PQ passes through the fixed point (-a, b) then the locus of T is

(A) $ay = 2b(x - b)$ (B) $bx = 2a(y - a)$ (C) $by = 2a(x - a)$ (D) $ax = 2b(y - b)$

Sol.



chord of contact PQ: $T = 0$.

$\Rightarrow y(k) = 2a(x+h)$.

$\Rightarrow ky = 2a(x+h)$.

chord passes through $(-a, b)$

$\Rightarrow kb = 2a(a+h)$

locus $(x \to h, y \to k)$ $\boxed{by = 2a(x-a)}$

7. If the tangent at the point P (X_1, Y_1) to the parabola $y^2 = 4ax$ meets the parabola $y^2 = 4a(x + b)$ at Q & R, then the mid point of QR is

(A) $(X_1 + b, Y_1 + b)$ (B) $(X_1 b, Y_1 - b)$ (C) (X_1, Y_1) (D) $(X_1 + b, Y_1)$

Sol.

eq^n of chord of $y^2 = 4a(x+b)$ with (h,k) as a middle point

$T = S_1$

$ky - 2a(x+h) - 4ab = k^2 - 4a(h+b)$.

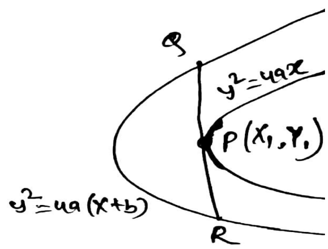
$\Rightarrow ky - 2ax - 2ah = k^2 - 4ah$

$\Rightarrow ky - 2ax = k^2 - 2ah$ —(i)

eq^n of tengent on $y^2 = 4ax$ at point (X_1, Y_1).

$yY_1 = 2a(x + X_1)$

$\Rightarrow yY_1 - 2ax = 2aX_1$. —(ii)



eq^n (i) and (ii) are same.

$\Rightarrow \frac{k}{Y_1} = \frac{-2a}{-2a} = \frac{k^2 - 2ah}{2aX_1}$

$\Rightarrow k = Y_1$,

and $2aX_1 = k^2 - 2ah \Rightarrow 2aX_1 = Y_1^2 - 2ah$

$\Rightarrow 2ah = 4aX_1 - 2aX_1 = 2aX_1$

$h = X_1$

$\Rightarrow h = X_1, \quad k = Y_1$

8. Two parabolas $y^2 = 4a(x - I_1)$ and $x^2 = 4a(y - I_2)$ always touch one another, the quantities I_1 and I_2 are both variable. Locus of their point of contact has the equation

(A) $xy = a^2$ (B) $xy = 2a^2$ (C) $xy = 4a^2$ (D) none

Sol. if parabola touch each other at (h, k)

eq^n of tangent on $y^2 = 4a(x - I_1)$ at (h, k)

$y \cdot k = 4a\left(\frac{x+h}{2}\right) - 4aI_1$

$yk - 2ax = 2ah - 4aI_1$ —(i)

eq^n of tangent on $x^2 = 4a(y - I_2)$ at (h, k).

$x \cdot h = 4a\left(\frac{y+k}{2}\right) - 4aI_2$

$\Rightarrow 2ay - xh = 4aI_2 - 2ak$ —(ii)

eq^n (i) and (ii) are same

$\Rightarrow \frac{k}{2a} = \frac{2a}{h} \Rightarrow hk = 4a^2$

Locus: $x \to h,\ y \to k$

$\Rightarrow xy = 4a^2$

9. Which of the following is the common tangent to the ellipses

$\frac{x^2}{a^2+b^2} + \frac{y^2}{b^2} = 1$ & $\frac{x^2}{a^2} + \frac{y^2}{a^2+b^2} = 1$?

(A) $ay = bx + \sqrt{a^4 - a^2b^2 + b^4}$ (B) $by = ax - \sqrt{a^4 + a^2b^2 + b^4}$

(C) $ay = bx - \sqrt{a^4 + a^2b^2 + b^4}$ (D) $by = ax - \sqrt{a^4 - a^2b^2 + b^4}$

Sol. Let $y = mx + c$ is common tangent

(i) $y = mx + c$ is tangent to $\frac{x^2}{a^2+b^2} + \frac{y^2}{b^2} = 1$

$\Rightarrow c^2 = (a^2+b^2)m^2 + b^2$ —(i)

(ii) $y = mx + c$ is tangent to $\frac{x^2}{a^2} + \frac{y^2}{a^2+b^2} = 1$

$\Rightarrow c^2 = a^2m^2 + a^2 + b^2$ —(ii)

$\Rightarrow$ from (i) & (ii) $m^2 = \frac{a^2}{b^2}$. and $c = \frac{\sqrt{a^4 + a^2b^2 + b^4}}{b}$

$\Rightarrow$ tangent is $by = \pm ax \pm \sqrt{a^4 + a^2b^2 + b^4}$

10. The locus of the mid points of the chords passing through a fixed point (α , ß) of the hyperbola,

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is

(A) a circle with centre $\left(\frac{\alpha}{2},\frac{\beta}{2}\right)$ (B) an ellipse with centre $\left(\frac{\alpha}{2},\frac{\beta}{2}\right)$

(C) a hyperbola with centre $\left(\frac{\alpha}{2},\frac{\beta}{2}\right)$ (D) straight line through $\left(\frac{\alpha}{2},\frac{\beta}{2}\right)$

Sol. Let mid point of chord is (h,k).

eq^n of chord is $T = S_1$.

$$\frac{xh}{a^2}-\frac{yk}{b^2}-1=\frac{h^2}{a^2}-\frac{k^2}{b^2}-1$$

$$\Rightarrow \frac{h^2}{a^2}-\frac{xh}{a^2}-\left(\frac{k^2}{b^2}-\frac{yk}{b^2}\right)=0$$

$$\Rightarrow \frac{\left(h-\frac{x}{2}\right)^2}{a^2}-\frac{\left(k-\frac{y}{2}\right)^2}{b^2}=\frac{x^2}{4a^2}-\frac{y^2}{4b^2}$$

passes through (α,β)

$$\Rightarrow \frac{\left(h-\frac{\alpha}{2}\right)^2}{p^2}-\frac{\left(k-\frac{\beta}{2}\right)^2}{q^2}=1$$

locus : $x\to h,\ y\to k$

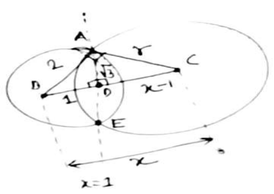
$$\frac{\left(x-\frac{\alpha}{2}\right)^2}{p^2}-\frac{\left(y-\beta/2\right)^2}{q^2}=1$$

Hyperbola with centre $(\alpha/2, \beta/2)$

11. The radius of circle S_2 which intersects the circle $S_1: x^2+y^2=4$ orthoganally and line $x=1$ is the comman chord of both the circle is

(A) 4 (B) $\sqrt{3}$ (C) $2\sqrt{3}$ (D) 2

Sol.



Let distance between centre of circles is x intersection piont of circle => common chord solving $x=1$ with $x^2+y^2=1$

=> A = $(1,\sqrt{3})$ and E $(1,-\sqrt{3})$

=> AE = $2\sqrt{3}$

=> AD = DE = $\sqrt{3}$

=> BD = $\sqrt{AB^2 - AD^2} = 1$

since circle are orthoganal => $2^2 + r^2 = x^2$ -(i)

and $\Delta AOC, (\sqrt{3})^2 + (x-1)^2 = r^2$ -(ii)

from (i) and (ii) $r = 2\sqrt{3}$

12. The mirror image of the focus to the parabola $4(x + y) = y^2$ w.r.t. the directrix is

(1) (0, 2) (2) (2, 2) (3) (-4, 2) (4) (-2, 2)

Sol.

13. If the parabola $y = (a - b)x^2 + (b - c)x + (c - a)$ touches the x-axis then the line $ax + by + c = 0$ always passes through a fixed point is

(A) (1, 2) (B) (-2, 1) (C) (2, 1) (D) (2, -1)

Sol. Put y = 0

$(a - b)x^2 + (b - c)x + (c - a) = 0$

Since the parabola touches the x-axis, so

D = 0

=> $(b - c)^2 - 4(a - b)(c - a) = 0$

=> $b^2 + c^2 - 2bc - 4(ac - a^2 - bc + ab)$

=> $4a^2 + b^2 + c^2 + 2bc - 4ac - 4ab = 0$

=> $(b + c)^2 + (2a)^2 - 2.2a(b + c) = 0$

=> $(b + c - 2a)^2 = 0$

=> $b + c - 2a = 0$

=> $-2a + b + c = 0$

=> $ax + by + c = 0$

=> Point is (-2, 1)

14. The number of real solutions of the equation $4x^{99} + 5x^{98} + 4x^{97} + 5x^{96} + ... + 4x + 5 = 0$ is

(1) 1 (2) 5 (3) 7 (4) 97

Sol. $4x^{99} + 5x^{98} + 4x^{97} + 5x^{96}$ $x^{98}(4x + 5) + x^{96}(4x + 5) + x^{94}(4x + 5) + ... + 1(4x + 5)$

$(4x + 5)(x^{98} + x^{96} + ... + 1)$ $x = \frac{-5}{4}$ $x^{98} + x^{96} + ... + 1 \neq 0$

Only one solution

15. STATEMENT-1 : Let AABC be right angle with vertices A(0,2), B(1,0) and C(0,0) If D is the point on AB such that the segment CD bisects angle C then the length of CD is $\frac{2\sqrt{2}}{3}$

STATEMENT-2: The number of points on the straight line which joins (-4, 11) to (16, -1) whose co-ordinates are positive integer is 3.

STATEMENT-3: If k = 2 then the lines L_1: $2x + y - 3 = 0$ L_2: $5x + ky - 3 = 0$ and L_3 : $3x - y - 2 = 0$ are concurrent.

(1) FTF (2) TFT (3) TTF (4) FFT

16. The equation $2x^2 + 3y^2 - 8x - 18y + 35 = K$ represents

(1) No locus if K > 0 (2) An ellipse if K< 0 **(3)** A point if K = 0 (4) A hyperbola if K > 0

Sol. By complete squaring method $2(x - 2)^2 + 3(y - 3)^2 = k$

If k = 0

$2(x-2)^2 + 3(y - 3)^2 = 0$

Then necessarily $(x - 2)^2 = 0$ & $(y - 3)^2 = 0$

Equation represents a point if k = 0

17. If $\frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b}$ then $a^a b^b c^c$

(A) 1 (B) abc (C) $\frac{1}{abc}$ (D) -1

So. $\log a = K(b-c) => a = e^{K(b-c)}$

$\log b = K(c-a) => b = e^{K(c-a)}$

$\log c = K(a-b) => c = e^{K(a-b)}$

$a^a b^b c^c = e^{K(ab-ac)} e^{K(bc-ba)} e^{K(ac-bc)}$

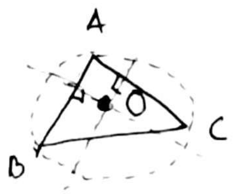
$= 1$

18. The equation of perpendicular bisector of two sides AB and AC of a tringle ABC are $x + y + 1 = 0$ and $x - y + 1 = 0$ respectively. If circumradius of ABC is 2 units, then the locus of vertex A is

(A) $x^2 + y^2 + 2x - 3 = 0$ (B) $x^2 + y^2 + 2x + 3 = 0$

(C) $x^2 + y^2 - 2x + 3 = 0$ (D) $x^2 + y^2 - 2x - 3 = 0$

Sol. Centre of circle will be intersection point of perpendicular bisecters of sides



$O \equiv (-1,0)$

let A = (h, K) => $(h + 1)^2 + K^2 = 2^2$ => $h^2 + k^2 + 2h - 3 = 0$

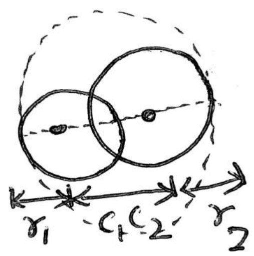
19. Radius of smallest circle which contains both the circle $x^2 + y^2 = 1$ and $(x - 1)^2 + (y - 2)^2 = 4$ is

(A) $\sqrt{5}+3$ (B) $\frac{\sqrt{5}+\sqrt{3}}{2}$ (C) $\frac{\sqrt{5}+3}{2}$ (d) $\frac{\sqrt{3}+5}{2}$

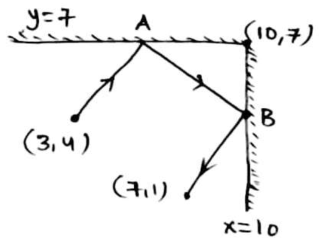
Sol. Diameter of smallest circle will be $= r_1 + c_1c_2 + r_2$

$r = \frac{1+\sqrt{5}+2}{2}$

$= \frac{\sqrt{5}+3}{2}$

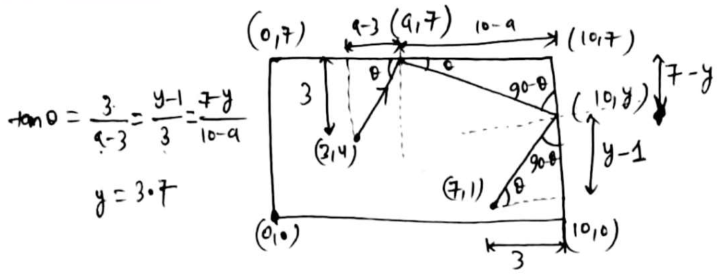


20. Two mirrors are placed along the line x = 10 and y = 7 as shown in figure. A ray of light originates from (3, 4) and come back through the point (7, 1). Then the g - coordinate of point B will be



(A) 3.7 (B) 3.8 (C) 3.9 (D) 4

Sol.



Section - B

1. The first term of arithmetic progression is 1 and the sum of the first nine terms equal to 369. The first and the ninth term of a geometric progression conicide with the first and the ninth term of the arithmetic. progression. Find the seventh term of the geometric progression.
Sol.

2. The third term of an A.P. is 18, and the seventh term is 30 ; find the sum of 17 terms.

3. The minimum value of the sum of real numbers a^{-5}, a^{-4}, $3a^{-3}$, 1, a^{8} and a^{10} with $a > 0$ is

4. From the point (4, 6) a pair of tangent lines are drawn to the parabola, $y^2 = 8x$ The area of the triangle formed by these pair of tangent lines & the chord of contact of the point (4, 6) is

5. An ellipse is drawn with major and minor axes of lengths 10 and 8 respectively. Using one focus as centre, a circle is drawn that is tangent to the ellipse, with no part of the circle being outside the ellipse. The radius of the circle is

6. The number of points on the ellipse $\frac{x^2}{50}+\frac{y^2}{20}=1$ from which pair of perpendicular tangents are drawn to ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$, is____.

Sol. For ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$, equation of director circle is $x^2 + y^2 = 25$
This director circle will cut the ellipse
$\frac{x^2}{50}+\frac{y^2}{20}=1$ at 4 points

7. Minimum value of $9\sec^2 x + 16\operatorname{cosec}^2 x$ is
Sol. $y = 9\sec^2 x + 16\operatorname{cosec}^2 x$
$= 9 + 9\tan^2 x + 16 + 16\cot^2 x$
$= 25 + 9\ 9\tan^2 x + 16\cot^2 x = 25 + (3\tan x \pm 4\cot x)^2 = 24$
$\Rightarrow y = (25 \pm 24) + (3\tan x \pm 4\cot x)^2$
Case I
$\Rightarrow y_{min} = 49$, when $3\tan x - 4\cot x = 0$
$\Rightarrow \tan x = \pm\sqrt{4/3}$
Case II
$\Rightarrow y_{min} = 1$ when $3\tan x + 4\cot x = 0 \Rightarrow \tan^2 x = -4/3$ which is not possible
Hence minimum value 49

8. $4\left(\cos^6\frac{\pi}{16}+\cos^6\frac{3\pi}{16}+\cos^6\frac{5\pi}{16}+\cos^6\frac{7\pi}{16}\right)=$_____.

Sol. since $\theta = \frac{\pi}{16} \Rightarrow 8\theta = \frac{\pi}{2}$

=> $\cos^6 5\theta = \cos^6(8\theta - 3\theta) = \cos^6\left(\frac{\pi}{2} - 3\theta\right) \sin^6 3\theta$

similarly, $\cos^6 7\theta = \sin^6 \theta$

=> $\cos^6\theta + \cos^6 3\theta + \cos^6 5\theta + \cos^6 7\theta = (\sin^6\theta + \cos^6\theta) + (\sin^6\theta + \cos^6 3\theta)$

=> $(\sin^6\theta + \cos^6\theta) = (\sin^2\theta + \cos^2\theta)^3 - 3\sin^2\theta\cos^2\theta(\sin^2\theta + \cos^2\theta)$

=> $(\sin^6\theta + \cos^6 3\theta) = (\sin^2 3\theta + \cos^2 3\theta)^3 - 3\sin^2 3\theta\cos^2 3\theta(\sin^2 3\theta + \cos^2 3\theta)$

=> $2 - \frac{3}{4}\sin^2 2\theta - \frac{3}{4}\sin^2 6\theta \Rightarrow 2 - \frac{3}{4}(\sin^2 2\theta + \cos^2 2\theta)$

=> $\frac{5}{4}$

9. If $\log_8^x + \log_4^{y^2} = 5$ and $\log_8^y + \log_4^{x^2} = 7$ then $\frac{x}{y} =$

Sol. $\frac{\log_2^x}{3} + \log_2^y = 5$

$\frac{a}{3} + b = 5 \Rightarrow a + 3b = 15$ - (i)

$\frac{\log_2^x}{3} + \log_2^y = 7$

$a + \frac{b}{3} = 7 \Rightarrow 3a + b = 21$ - (ii)

from (i) & (ii) $a = 6, b = 3$

$\log_2^x = 6 \Rightarrow x = 2^6$ & $\log_2^y = 3 \Rightarrow y = 8$

10. The minimum area of the triangle formed by the tangent to the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$, and theco-ordinate axes is

Sol. Tangent at P is $\frac{x}{4}\cos\theta + \frac{y}{3}\sin\theta = 1$