PHYSICS

SECTION 1 (Maximum Marks: 12)

- This section contains FOUR (04) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : -1 In all other cases.

1. A particle of mass m is moving in the xy-plane such that its velocity at a point (x, y) is given as $\vec{v} = \alpha(y\hat{x} + 2x\hat{y})$, where α is a non-zero constant. What is the force $\vec{F}$ acting on the particle?

(a) $\vec{F} = 2m\alpha^2(x\hat{x} + y\hat{y})$ (b) $\vec{F} = m\alpha^2(y\hat{x} + 2x\hat{y})$

(c) $\vec{F} = 2m\alpha^2(y\hat{x} + x\hat{y})$ (d) $\vec{F} = m\alpha^2(x\hat{x} + 2y\hat{y})$

Sol.

$$\vec{v} = \alpha(y\hat{x} + 2x\hat{y})$$

$$\Rightarrow \vec{v}_x = \alpha y\hat{x} \quad \& \quad \vec{v}_y = 2\alpha x\hat{y}$$

$$\Rightarrow \vec{a}_x = \alpha(v_y)\hat{x} \quad \Big| \quad \vec{a}_y = 2\alpha(v_x)\hat{y}$$

$$\Rightarrow \vec{a}_x = 2\alpha^2 x\hat{x} \quad \Big| \quad \vec{a}_y = 2\alpha^2 y\hat{y}$$

$$\Rightarrow \vec{a} = 2\alpha^2[x\hat{x} + y\hat{y}]$$

$$\Rightarrow \boxed{\vec{F} = 2m\alpha^2[x\hat{x} + y\hat{y}]}$$

2. A particle moves in a straight line with a retardation proportional to its displacement. Its loss of kinetic energy for any displacement x is proportional to

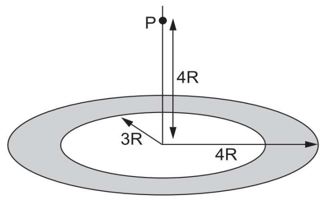
(a) - x **(b)** - x^2 (c) $\ln x$ (d) e^x

$$a = -kx \Rightarrow \frac{dv}{dt} = -kx.$$

$$\frac{dv}{dx} \times \frac{dx}{dt} = -kx \Rightarrow vdv = -kxdx$$

$$\int_{v_i}^{v_f} v dv = -k\int_0^x x dx \Rightarrow \Delta KE \propto -x^2$$

3. A thin uniform annular disc (see figure) of mass M has outer radius 4R and inner radius 3R. The work required to take a unit mass from point P on its axis to infinity is



(a) $\frac{2GM}{7R}\left(4\sqrt{2}-5\right)$ (b) $\frac{2GM}{7R}\left(4\sqrt{2}+5\right)$

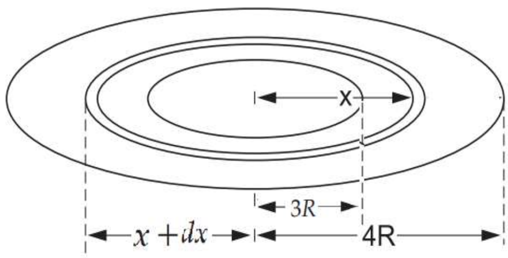
(c) $\frac{GM}{4R}$ (d) $\frac{2GM}{5R}\left(\sqrt{2}+5\right)$

Sol. Mass per unit area $\sigma = \frac{M}{\pi[(4R)^2-(3R)^2]}$,

area of ring $= 2\pi x dx$

mass of ring $= dm = 2\pi\sigma x dx$

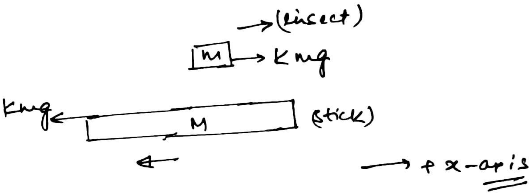
$$V_P = -G\int_{3R}^{4R}\frac{dm}{[(4R)^2+x^2]^{1/2}}$$



4. An insect of mass m is initially at one end of a stick of length L and mass M, which rests on a smooth floor. The coefficient of friction between the insect and the stick is k. The minimum time in which the insect can reach the other end of the stick is t. Then t^2 is equal to

(a) 2L/kg (b) 2Lm/kg(M + m) **(c)** 2LM/kg(M + m) (d) 2Lm/kgM

Sol.



For insect :-

$$Kmg = m\frac{d^2 x_{i|g}}{dt^2}$$

$$\vec{x}_{i|g} = \frac{Kgt^2}{2}$$

$$-Kmg = M\left(\frac{d^2 x_{s|g}}{dt^2}\right) \Rightarrow (x_{s|g}) = -\frac{Kmg}{M}\frac{t^2}{2}$$

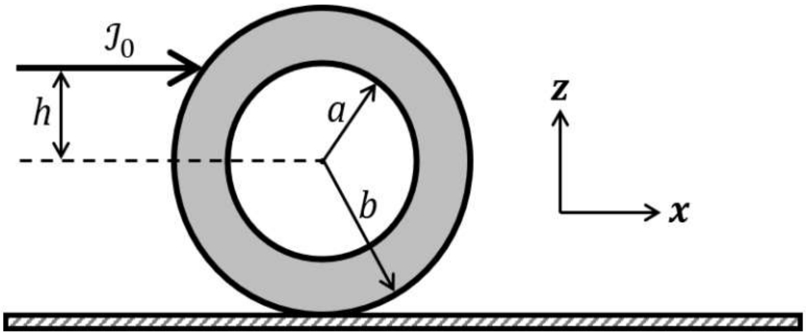
$$\vec{x}_{i|s} = \vec{x}_{i|g} - \vec{x}_{s|g}$$

$$\Rightarrow \quad l = \frac{Kgt^2}{2} + \frac{Kmgt^2}{2M} \Rightarrow t^2 = \frac{2Ml}{(m+M)Kg}$$

SECTION 2 (Maximum Marks: 12)

- This section contains THREE (03) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 ONLY if (all) the correct option(s) is(are) chosen;
  Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;
  Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;
  Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;
  Zero Marks : 0 If unanswered;
  Negative Marks : -2 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option(s) (i.e. the question is unanswered) will get 0 marks and
  choosing any other option(s) will get -2 marks.

5. An annular disk of mass M, inner radius a and outer radius b is placed on a horizontal surface with coefficient of friction μ, as shown in the figure. At some time, an impulse $\mathcal{J}_0\hat{x}$ is applied at a height h above the center of the disk. If $h = h_m$ then the disk rolls without slipping along the x-axis. Which of the following statement(s) is(are) correct?



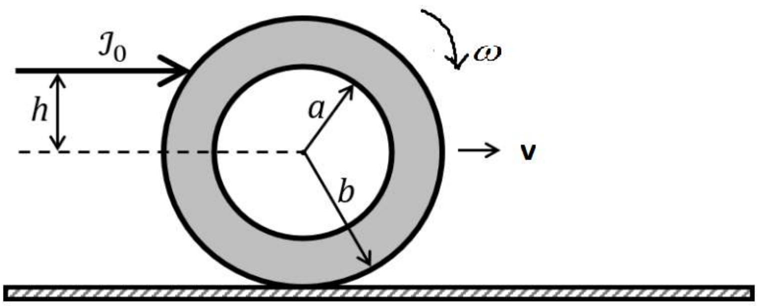
(a) For $\mu \neq 0$ and $a \to 0$, $h_m = b/2$.

(b) For $\mu \neq 0$ and $a \to b$, $h_m = b/2$.

(c) For $h = h_m$, the initial angular velocity depend on the inner radius a.

(d) For $\mu = 0$ and $h = 0$, the wheel always slides without rolling.

Sol.



$\tau_0 = Mv$ eq^n - (i)

and $\tau_0 h = \frac{M}{2}(a^2 + b^2)\omega$

If $h = h_m \Rightarrow \omega = \frac{v}{b}$

$\tau_0 h_m = \frac{M}{2}(a^2 + b^2)\frac{v}{b}$

$\Rightarrow$ using eq^n (i)

$h_m = \frac{a^2 + b^2}{2b}$

(a) If $a \to 0 \quad h_m \to \frac{b}{2}$ (b) $a \to b \quad h_m \to b$

(c) $h = h_m$ it is the case of pure rolling motion (as given) hence initial velocity will not depend upon inner radius 'a'

(d) for $\mu = 0$, $h = 0$ it's the case sliding motion

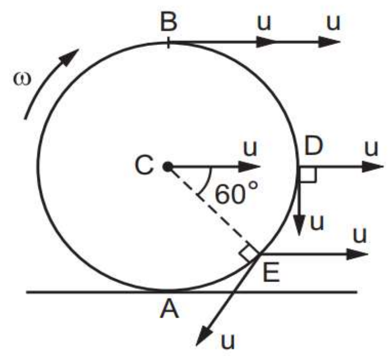
6. A ring rolls without slipping on the ground. Its centre C moves with a constant speed u. P is any point on the ring. The speed of P with respect to the ground is v

(a) $0 \le v \le 2u$ (b) $v = u$, if CP is horizontal.

(c) $v = u$, if CP makes an angle of 60° with the horizontal and P is below the horizontal level of C.

(d) $v = \sqrt{2}u$, if CP is horizontal.

Sol. Every point on the ring has a horizontal velocity u due to its linear motion, and in addition a velocity u, tangential to the ring, due to its rotational motion. The resultant of these two is the velocity of the point with respect to the ground.



Hence,

$$v_A = 0,\ v_B = 2u,\ v_D = \sqrt{2}u,\ v_E = u.$$

7. A satellite is revolving around earth in a circular orbit at a height $\frac{R}{2}$ from the surface of earth. Which of the following are correct statements about it (M = mass of earth, R = radius of earth, m = mass of satellite)?

(a) Its orbital velocity is $\sqrt{\frac{2GM}{3R}}$ **(b)** Its total energy is $-\frac{2GMm}{3R}$

(c) Its kinetic energy is $\frac{GMm}{3R}$ **(d)** Its potential energy is $-\frac{2GMm}{3R}$

Sol. Orbital velocity $= \sqrt{\frac{G}{R+h}} = \sqrt{\frac{GM}{R+\frac{R}{2}}} = \sqrt{\frac{2GM}{3R}}$

Total energy $= -\frac{GMm}{2\times\frac{3R}{2}} = -\frac{GMm}{3R}$

Kinetic energy $= \frac{1}{2}m\left\{\frac{2GM}{3R}\right\} = \frac{GMm}{3R}$

Potential energy = Total energy – Kinetic energy

$= -\frac{GMm}{3R} - \frac{GMm}{3R}$

$= -\frac{2GMm}{3R}$

SECTION 3 (Maximum Marks: 24)

- This section contains SIX (06) questions.
- The answer to each question is a NON-NEGATIVE INTEGER.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 If ONLY the correct integer is entered;
  Zero Marks : 0 In all other cases.

8. A satellite is in a circular orbit very close to the surface of a planet. At some point it is given an impulse along its direction of motion, causing its velocity to increase η times. It now goes into an elliptical orbit, with the planet at the centre of the ellipse. The maximum possible value of η for this to occur.
Calculate $4\eta^2$ =________.

Sol. The initial velocity of the satellite is given by $v = \sqrt{\frac{GM}{R}}$.

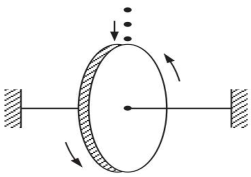
The satellite will remain in the elliptical orbit as long as $v < v_e$, where v_e is the escape velocity.

Also, $v_e = \sqrt{\frac{2GM}{R}}$.

$\therefore \quad \eta v \le v_e \quad$ or $\quad \eta \cdot \sqrt{\frac{GM}{R}} \le \sqrt{\frac{2GM}{R}}$

or $\quad \eta \le \sqrt{2} \quad$ or $\quad \eta_{max} = \sqrt{2}$.

9.



A disc of mass $m_0 = 10$ kg rotates freely about a fixed horizontal axis passing through its centre. A thin cotton pad is fixed to its rim, which can absorb water. The mass of water dripping onto the pad per unit time is $\mu = 0.5$ kg/min. If t be the time will the angular velocity of the disc get reduced to half its initial value.
Calculate t = _____Sec.

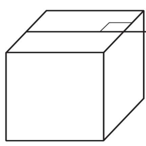
Sol. $L = I_0\omega_0 = \frac{I\omega_0}{2} \quad$ or $\quad 2I_0 = I$

or $\quad 2\left(\frac{1}{2}m_0r^2\right) = \frac{1}{2}m_0r^2 + (\mu t)r^2$

or $\quad t = \frac{m_0}{2\mu}$.

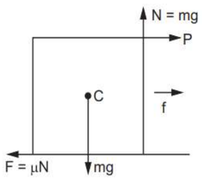
$$\Rightarrow t = \frac{10}{2\times 0.5} = 600 \text{ Sec}$$

10.



A solid cube is placed on a horizontal surface. The coefficient of friction between them is $\mu = 0.2$. A variable horizontal force perpendicular to one edge and passing through the midpoint of that edge is applied on the cube's upper face. The maximum acceleration with which it can move without toppling is____m/sec^2 (Take g = 10 m/sec^2)

Sol.



Let a be the length of each edge of the cube.

At the position of toppling, taking the torque about C, we have

$$\left(\mu mg \times \frac{a}{2}\right) + \left(P \times \frac{a}{2}\right) = mg \times \frac{a}{2}$$

or $\quad P = mg - \mu mg.$

Let f = acceleration.

$$P - \mu mg = mf$$

or $\quad mg - 2\mu mg = mf \quad$ or $\quad f = g(1 - 2\mu).$

11. A point mass of 1 kg collides elastically with a stationary point mass of 5 kg. After their collision, the 1 kg mass reverses its direction and moves with a speed of $2ms^{-1}$. If $|\vec{p}|$ be the magnitude of the total momentum of the system after the collision and $|k|$ be the magnitude of the KE of the COM.

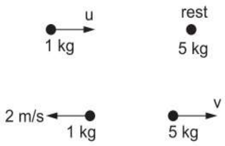
Calculate $|\vec{p}| + 100 \times |k|$ =______.

Sol. $u = -2 + 5v$

$v + 2 = -1(0 - u) = u$

Solving, $u = 3$ m/s, $\quad v = 1$ m/s

KE of CM $= \frac{1}{2}(6 \text{ kg})(0.5 \text{ m/s})^2 = 0.75$ J.



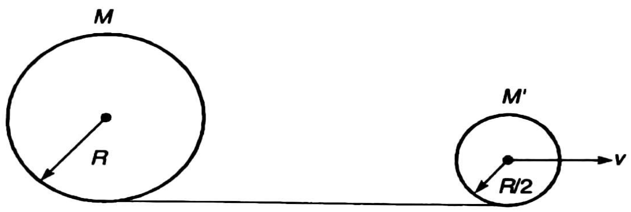
12. A carpet of mass M = 50 kg made of inextensible material is rolled along its length in the form of a cylinder of radius R = 3m and is kept on a rough floor. The carpet starts unrolling without sliding on the floor when a negligibly small push is given to it. If v be the horizontal velocity of the axis of the cylindrical part of the carpet when its radius reduces to R/2. Calculate v^2 =______m^2/sec^2. (Take $g = 10$ m/sec^2)

Sol. **Let M' be the mass of unwound carpet. Then,**

$$M' = \left(\frac{M}{\pi R^2}\right)\pi\left(\frac{R}{2}\right)^2 = \frac{M}{4}$$

From conservation of mechanical energy :

$$MgR - M'g\frac{R}{2} = \frac{1}{2}\left(\frac{M}{4}\right)v^2 + \frac{1}{2}I\omega^2$$



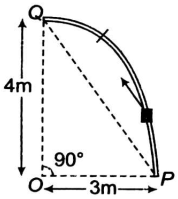
or $\quad MgR - \left(\frac{M}{4}\right)g\left(\frac{R}{2}\right)$

$$= \frac{Mv^2}{8} + \frac{1}{2}\left(\frac{1}{2} \times \frac{M}{4} \times \frac{R^2}{4}\right)\left(\frac{v}{R/2}\right)^2$$

or $\quad \frac{7}{8}MgR = \frac{3Mv^2}{16}$

$\therefore \quad v = \sqrt{\frac{14Rg}{3}}$

13. Consider an elliptically shaped rail PQ in the vertical plane with OP = 3 m and OQ = 4 m. A block of mass 1 kg is pulled along the rail from P to Q with a force of 18 N, which is always parallel to line PQ (see figure). Assuming no frictional losses, the kinetic energy of the block when it reaches Q is $(n \times 10)$ J. The value of n is ______. (Take $g = 10 m/sec^2$)



Sol. From work-energy theorem.

$$W_F + W_{mg} = K_f - K_i$$
$$10\times5 - 40 = K_f$$

SECTION 4 (Maximum Marks: 12)

- This section contains TWO (02) paragraphs.
- Based on each paragraph, there are TWO (02) questions.
- For each question, enter the correct options on OMR sheet
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 If ONLY the correct option is choosen.
  Zero Marks : 0 In all other cases.

A ball is thrown with a speed $u = 10\sqrt{3}$ m/s at an angle θ so that the area enclosed by the balls trajectory with the horizontal surface is maximum. Taking $g = 10$ m/s^2 and $\sqrt{3} = 1.732$

14. Calculate the angle θ so that area enclosed by the balls trajectory with the horizontal surface is maximum. θ = ______°.

A ball is thrown with a speed $u = 10\sqrt{3}$ m/s at an angle θ so that the area enclosed by the balls trajectory with the horizontal surface is maximum. Taking $g = 10$ m/s^2 and $\sqrt{3} = 1.732$

15. Calculate maximum horizontal surface area = _______ m^2.

Sol. Coordinates of the ball at t = t is

$x = u\cos\theta t$

$y = u\sin\theta t - \frac{1}{2}gt^2$

and time of flight is $\frac{2u\sin\theta}{g}$

Area under trajectory is

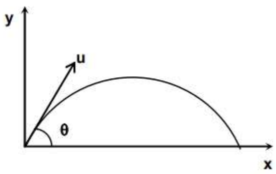
$$A = \int_0^x y\,dx = \int_0^{\frac{2u\sin\theta}{g}} \left(u\sin\theta t - \frac{gt^2}{2}\right) u\cos\theta\, dt$$

$$\therefore\ A = \frac{2u^4}{3g^2}\sin^3\theta\cos\theta$$

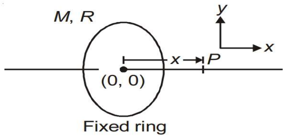
For A to be maximum, $\frac{dA}{d\theta} = 0$

$\Rightarrow \theta = 60°$

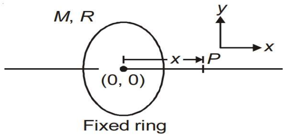
$$\therefore\ A_{max} = \frac{\sqrt{3}u^4}{8g^2} = 194.85 \approx 195\ m^2$$



The gravitational force $\vec{F}$ upon an object of mass m placed in external gravitational field $\vec{E}_{ext}$ is given by $\vec{F} = m\vec{E}_{ext}$. Consider a fixed mass ring (of mass M, radius R) in y-z plane, with centre at (0, 0) and axis of ring along x direction.



16. The intensity of gravitational field has the maximum value at x equal to $\pm$ _____R.



The gravitational force $\vec{F}$ upon an object of mass m placed in external gravitational field $\vec{E}_{ext}$ is given by $\vec{F} = m\vec{E}_{ext}$. Consider a fixed mass ring (of mass M, radius R) in y-z plane, with centre at (0, 0) and axis of ring along x direction.

17. The value of maximum gravitational field intensity is ____G.

Sol.

$$I = \frac{Gmx}{(R^2 + x^2)^{3/2}}$$

For maximum,

$$\frac{dI}{dx} = 0$$

$$\Rightarrow x = \pm\frac{R}{\sqrt{2}}$$

$$I_{max} = \frac{G2m}{3\sqrt{3}R^2}$$

CHEMISTRY

SECTION 1 (Maximum Marks: 12)

- This section contains FOUR (04) questions.
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- For each question, choose the option corresponding to the correct answer.
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1. Equivalent mass of H_3PO_2 when it undergoes disproportionation to PH_3 and H_3PO_3 will be _____.

(a) $\frac{M.W}{4}$ (b) $\frac{4 \times M.W}{3}$ (c) $\frac{M.W}{24}$ **(d)** $\frac{3 \times M.W}{4}$

2. On the appreciable hydrolysis of a salt of strong acid and weak base to be used to calculate degree of hydrolysis 'X'

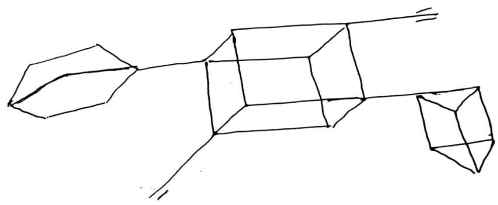
(a) $x = \sqrt{\frac{K_w}{K_h.K_a}}$ (b) $x = \sqrt{\frac{K_w}{K_{b}\cdot_{a}}}$ (c) $x = \sqrt{\frac{K_w}{K_a.K_b}}$ **(d)** None of these

Sol.

3. De Broglie wavelength of an electron travelling with speed equal to 1% of the speed of light ____.

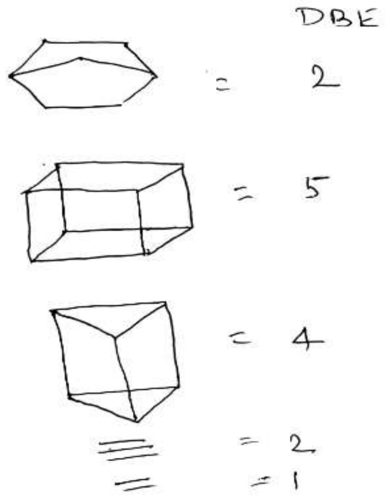
(a) 450 Pm (b) 210 Pm **(c)** 242 Pm (d) 320 Pm

4. Total double bond (DBE) equivalent present in compound



(a) 6 (b) 8 (c) 11 **(d)** 14

Sol.



SECTION 2 (Maximum Marks: 12)

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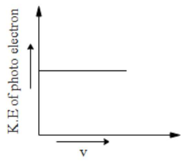
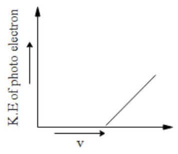
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks; choosing ONLY (A) and (D) will get +2marks;
  choosing ONLY (B) and (D) will get +2 marks; choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark; choosing ONLY (D) will get +1 mark;
  choosing no option(s) (i.e. the question is unanswered) will get 0 marks and
  choosing any other option(s) will get -2 marks.

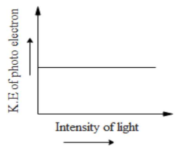
5. Which of the following statements is/are not correct for following compounds?
(i) $SCl_2(OCH_3)_2$ and (ii) $SF_2(OCH_3)_2$
(a) Cl-atom Occupy equatorial position in case of (i) and F-atoms occupy equatorial position in case of (ii)

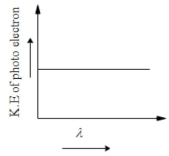
(b) OCH_3 groups in both cases occupy the same position
(c) Cl-atoms occupy axial position in case of (i) and F atoms occupy equatorial position in case of (ii)
(d) Cl and F-atoms occupy either exial or equatorial position in case of (i) and (ii) respectively

6. Which is/are correct graph?

(a) 

(b) 

(c) 

(d) 

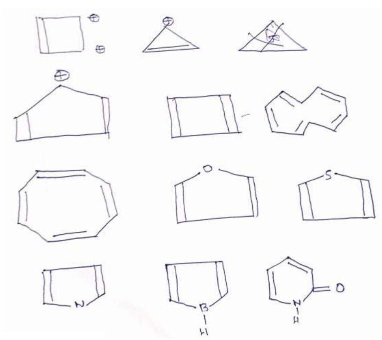
7. Aqueous solutions of HNO_3, KOH, CH_3COOH and CH_3COONa of identical concentrations are provided. The pair(s) of solutions which froms a buffer upon mixing is/are

(a) HNO_3 and CH_3COOH
(b) KOH and CH_3COONa
(c) HNO_3 and CH_3COONa
(d) CH_3COOH and CH_3COONa

SECTION 3 (Maximum Marks: 24)

- This section contains SIX (06) questions.
- The answer to each question is a NON-NEGATIVE INTEGER.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 If ONLY the correct integer is entered;
  Zero Marks : 0 In all other cases.

8. Among the given molecules identify aromatic molecules



ans : - 6

9. Given

$A_g \rightleftharpoons B_{(g)}$ $K_c = 10$
$B_g \rightleftharpoons C_{(g)}$ $K_c = 2$
$C_g \rightleftharpoons D_{(g)}$ $K_c = 0.01$

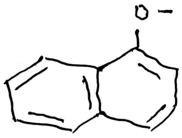
Calculate Kc for the reaction

$D_g \rightleftharpoons A_{(g)}$

Sol. ans:- 5

$$Kc = \frac{1}{10} \times \frac{1}{0.01} \times \frac{1}{2}$$

$= 5$

10. 

X = Total number of resonating structure value of = $\frac{x+2}{2}$ = ?

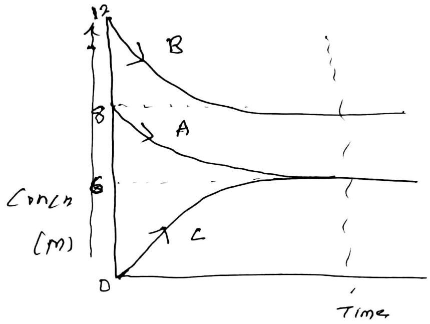
Sol. $\frac{10+2}{2}=\frac{12}{2}=6$

11. One mole of N_2H_4 loses 10 moles of e^- to form a new compound Y. Assuming that all the Nitrogen appears in the new compound, what is the oxidation state of Nitrogen in A?
[There is no change in the O.S of Hydrogen]

ans;- 3

12. Total number of structural isomerism in C_6H_{12} (only alicyclic compounds)

ans;- 12

13. 

The gaseous reaction $A(g) + n\,(B)_g \rightleftharpoons mc_{(g)}$

. $A_{(g)} + n\,B_{(g)} \rightleftharpoons mc(g)$ is

As represented in the above graph, what is the value of n + m = ?

ans;- 5

SECTION 4 (Maximum Marks: 12)

- This section contains TWO (02) paragraphs.
- Based on each paragraph, there are TWO (02) questions.
- For each question, enter the correct options on OMR sheet
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct option is choosen.
Zero Marks : 0 In all other cases.

We know that balancing of a chemical equation is entirely besed on Law of conservation of mass. However the concept of Principle of Atomic conservation (PoAC) can also be related to Law of conservation of mass in a chemical reaction so PoAC can also act as a technique for balancing a chemical equation e.g. for a chemical reaction

$ABC_3 \rightarrow AB + C_2$

on applying PoAC for A,B & C and relating the 3 equations we get

$$\frac{n_{ABC_3}}{2}=\frac{n_{AB}}{2}=\frac{n_{C_2}}{3}\left(n_x : number\ of\ moles\ of\ x\right)$$

thus the coefficients of ABC_3 , AB and C_2 in the balanced equation will be 2, 2 and 3 respectively and the balanced chemical equation can we represented as $2ABC_3 \rightarrow 2AB + 3C_2$

Based on above calculate the following

14. The numarical coefficients p, q, r in the balanced chemical equation $pA + qB_2 \rightarrow rA_2B_5$. Then $\frac{q}{p}$ = _____.

(a) 2p = r **(b)** q = 1.25p (c) r = 2q (d) q = 0.8p

We know that balancing of a chemical equation is entirely besed on Law of conservation of mass. However the concept of Principle of Atomic conservation (PoAC) can also be related to Law of conservation of mass in a chemical reaction so PoAC can also act as a technique for balancing a chemical equation e.g. for a chemical reaction

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thus the coefficients of ABC_3 , AB and C_2 in the balanced equation will be 2, 2 and 3 respectively and the balanced chemical equation can we represented as $2ABC_3 \rightarrow 2AB + 3C_2$

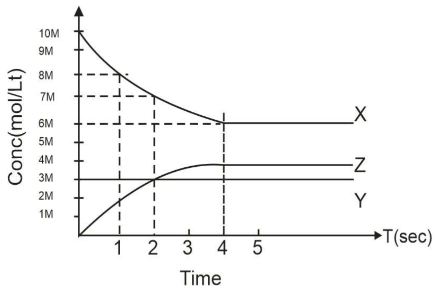
Based on above calculate the following

15. If the atomic masses of X and Y are 10 and 30 respectively than the mass of XY_3 fromed when 120g of Y_2 reacts completely with X in reaction

$X + Y_2 \rightarrow XY_3$

(a) 133.3g (b) 200g (c) 266.6g (d) 400g

X, Y, and Z react in the 1:1:1 stoichiometric ratio. The concentration of X, Y, and Z were found to vary with time as shown in the figure below



16. Following equilibrium reaction represents the variation of concentration with time. Each reaction has been

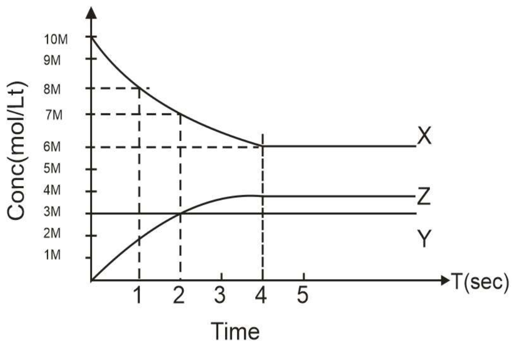
given an integeral value at the end of each reaction as [n], n = any integer . Enter the integeral value of the correct equilibrium reaction

(a) $X_{(g)} + Y_{(g)} \rightleftharpoons Z_{(g)}$ [1] (b) $X_{(g)} + Y_{(s)} \rightleftharpoons Z_{(g)}$ [2]

(c) $Z_{(g)} + Y_{(s)} \rightleftharpoons X_{(g)}$ [3] (d) $Z_{(g)} + X_{(g)} \rightleftharpoons Y_{(g)}$ [4]

Sol. Clearly concentration of Y is not changing with time hence it will be pure solid or liquid. Concentration of X is decresing hence it will be reactant and Z will be product of the reaction

X, Y, and Z react in the 1:1:1 stoichiometric ratio. The concentration of X, Y, and Z were found to vary with time as shown in the figure below



17. Value of the equilibrium constant 3 × (K_c) for the equilibrium represented the above graph will be

(a) $\frac{9}{2}$ (b) $\frac{21}{4}$ **(c)** $\frac{2}{3}$ (d) $\frac{12}{7}$

Sol. $K_c = \frac{\lfloor Z_{(g)} \rfloor_{eq}}{\lfloor X_{(g)} \rfloor_{eq}} = \frac{4}{6}$

MATHEMATICS

SECTION 1 (Maximum Marks: 12)

- This section contains FOUR (04) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : -1 In all other cases.

1. The equation of the locus of a point which moves so as to be at equal distances from the point (a, 0)

and the y-axis is

(a) $y^2 - 2ax + a^2 = 0$ (b) $y^2 + 2ax + a^2 = 0$ (c) $x^2 - 2ay + a^2 = 0$ (d) $x^2 + 2ay + a^2 = 0$

Sol. $(h-a)^2+k^2 = h^2$

$\Rightarrow -2ah + a^2 + k^2 = 0$ Replace (h, k) by (x, y), then $y^2 - 2ax + a^2 = 0$ the required locus.

2. Tangent to the parabola $y = x^2 + 6$ at (1, 7) touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ at the point

(a) (-6, 29) (b) (-13, -9) **(c)** (-6, -7) (d) (13, 7)

Sol. Equation of tangent at (1, 7) to $y = x^2 + 6$

$$\frac{1}{3}(y+7) = x.1+6 \Rightarrow y = 2x+5 \qquad eq^n \text{ ....(i)}$$

This tangent also touches the circle

$x^2 + y^2 + 16x + 12y + c = 0$ ....(ii)

Now solving (i) and (ii), we get

$x^2 + (2x + 5)^2 + 16x + 12(2x + 5) + c = 0 \Rightarrow 5x^2 + 60x + 85 + c = 0$

Since, roots are equal so

$b^2 - 4ac = 0 \Rightarrow (60)^2 - 4 \times 5 \times (85 + c) = 0$

$$85 + c = 180 \Rightarrow 5x^2 + 60x + 180 = 0 \Rightarrow x = \frac{60}{10} = -6$$

& y = -7

Hence, point of contact is (-6, -7)

3. The equation of an ellipse whose focus (-1, 1), whose directrix is $x - y + 3 = 0$ and whose eccentricity is $\frac{1}{2}$, is given by

(a) $7x^2 + 2xy + 7y^2 + 10x - 10y + 7 = 0$ (b) $7x^2 - 2xy + 7y^2 - 10x + 10y + 7 = 0$

(c) $7x^2 - 2xy + 7y^2 - 10x - 10y - 7 = 0$ (d) $7x^2 - 2xy + 7y^2 + 10x + 10y - 7 = 0$

Sol. Let any point on it be (x, y), then $\dfrac{\sqrt{(x+1)^2} + \sqrt{(y-1)^2}}{\dfrac{x-y+3}{\sqrt{2}}} = \dfrac{1}{2}$

Squaring and simplifiying, we get

$$7x^2 + 2xy + 7y^2 + 10x - 10y + 7 = 0$$

4. Find the coefficient of x^3 in the expansion of $(1 + x + 2x^2)\left(2x^2 - \frac{1}{3x}\right)^9$

(a) $-\frac{224}{9}$ (b) $-\frac{112}{27}$ **(c)** $-\frac{224}{27}$ (d) $-\frac{112}{9}$

SECTION 2 (Maximum Marks: 12)

- This section contains THREE (03) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 ONLY if (all) the correct option(s) is(are) chosen;
Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;
Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;
Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks : 0 If unanswered;
Negative Marks : -2 In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then

choosing ONLY (A), (B) and (D) will get +4 marks;
choosing ONLY (A) and (B) will get +2 marks;
choosing ONLY (A) and (D) will get +2marks;
choosing ONLY (B) and (D) will get +2 marks;
choosing ONLY (A) will get +1 mark;
choosing ONLY (B) will get +1 mark;
choosing ONLY (D) will get +1 mark;
choosing no option(s) (i.e. the question is unanswered) will get 0 marks and
choosing any other option(s) will get -2 marks.

5. Let A and B be two distinct points on the pa- rabola $y^2 = 4x$. If the axis of the parabola touches a circle of radius r having AB as its diameter, then the slope of the line joining A and B can be

(a) $-\frac{1}{r}$ (b) $\frac{1}{r}$ **(c)** $\frac{2}{r}$ **(d)** $-\frac{2}{r}$

Sol. $A(t_1^2, 2t_1)$ $B(t_2^2, 2t_2)$

Centre : $\left[\frac{t_1^2+t_2^2}{2}, t_1+t_2\right]$

$t_1 + t_2 = \pm r$

slope of chord = $\frac{2}{t_1+t_2} = \pm\frac{2}{r}$

6. In the expansion of $\left(\sqrt[3]{4}+\frac{1}{\sqrt[4]{6}}\right)^{20}$

A. the number of rational terms = 4 **B**. the number of irrational terms = 19
C. the middle term is irrational D. the number of irrational terms = 17

7. If the coefficient of the middle term in the expansion of $(1+x)^{2n+2}$ is coefficients of middle terms in the expansion of $(1+x)^{2n+1}$ are q and r, then. Which of the following options are wrong

(a) p + q = r **(b)** p + r = q (c) p = q + r **(d)** p + q + r = 0

Sol. Since (n + 2) th term is the middle term in the expansion of $(1+x)^{2n+2}$, therefore $p = {}^{2n+2}C_{n+1}$.
Since (n + 1) th and (n + 2) th terms are middle terms in the expansion of (1+ x)2n+1 , therefore $q = {}^{2n+1}C_n$
$r = {}^{2n+1}C_{n+1}$ But ${}^{2n+1}C_n + {}^{2n+1}C_{n+1} = {}^{2n+2}C_{n+1}$. q + r = p

SECTION 3 (Maximum Marks: 24)

- This section contains SIX (06) questions.
- The answer to each question is a NON-NEGATIVE INTEGER.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
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8. If the line $y = 3x + 1$ touches parabola $y^2 = kx$, then find k.

ans;- 12.00

9. If S_1, S_2, S_3 are the sums of first n natural numbers, their squares, their cubes respectively, then find the value of $\frac{S_3(1+8S_1)}{S_2^2}$.

ans;- 9.00

10. If a double ordinate of the parabola $y^2 = 4ax$ be of length 8a, then the angle (in degrees) between the lines joining the vertex of the parabola to the ends of this double ordinate is

ans;- 90

11. The coefficient of x^5 in the expansion of $(1+x^2)^5(1+x)^4$ is

Sol. We have $(1+x^2)^5(1+x)^4 = ({}^5C_0 + {}^5C_1x^2 + {}^5C_2x^4 + \ldots)$ $({}^4C_0 + {}^4C_1x + {}^4C_2x^2 + {}^4C_3x^3 + {}^4C_4x^4)$ So coefficient of x^5 in $[(1+x^2)^5(1+x)^4] = {}^5C_2 . {}^4C_1 + {}^4C_3 . {}^5C_1 = 60.$

ans ;- 60

12. A line passes through A(1,1) and B(100,1000). The number of points with integral co-ordinates on the line strictly between A and B is

Sol. The slope of the line is

$$\frac{1000-1}{100-1} = \frac{111}{11}$$

So, all the points will have the form (1+11t,1+111t)

$\Rightarrow 1 < 1+11t < 100 \Rightarrow 0 < 11t < 99 \Rightarrow 0 < t < 9$ ....(i)

and $1 < 1+111t < 1000 \Rightarrow 0 < 111t < 999 \Rightarrow 0 < t < 9$ ....(ii)

Hence from (i) and (ii), there are 8 such values of t and hence there are 8 such points.

ans :- 8

13. A piece of cheese is kept at P(1,2) in a plane . A rat sitting somewhere in the fourth quadrant with no sense of direction starts moving towards the cheese along the line x - y = 3, and after reaching a point Q(a,b), it starts getting farther from the cheese, then (a + b) is equal to

Sol. Let the point Q be $(h+3, h)$

Now since after reaching point Q it starts moving farther

So, PQ must be perpendicular to the line

Slope of PQ $\frac{h-2}{h+2}$

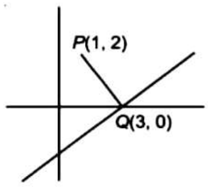
Slope of the line 1

$\Rightarrow h - 2 = -(h+2)$

$\Rightarrow 2h = 0 \Rightarrow h = 0$

So, $Q = (3, 0)$

$\therefore$ $a + b$ is 3



SECTION 4 (Maximum Marks: 12)

- This section contains TWO (02) paragraphs.
- Based on each paragraph, there are TWO (02) questions.
- For each question, enter the correct options on OMR sheet
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct option is choosen.

Zero Marks : 0 In all other cases.

Let P(x) be a quadratic polynomial with real coefficients such that for all real x the relation 2(1 + P(x) = P(x - 1) + P(x+1) holds.

If P(0)=8 and P(2)=32 then :

14. The sum of all the coefficient of P(x) is:

(a) 20 **(b)** 19 (c) 17 (d) 15

Let P(x) be a quadratic polynomial with real coefficients such that for all real x the relation
2(1 + P(x) = P(x - 1) + P(x+1) holds.
If P(0)=8 and P(2)=32 then :

15. If the range of P(x) is [m, 8), then the value of m is :
(a) -12 (b) 15 **(c)** -17 (d) -5

Two consecutive numbers from n natural numbers 1, 2, 3,....., n are removed. Arithmetic mean of the remaining numbers is $\frac{105}{4}$.

16. The value of n is:
(a) 48 **(b)** 50 (c) 52 (d) 49

Two consecutive numbers from n natural numbers 1, 2, 3,....., n are removed. Arithmetic mean of the remaining numbers is $\frac{105}{4}$.

17. The Squre of the G.M. of the removed numbers is :
(a) $\sqrt{30}$ (b) $\sqrt{42}$ **(c)** $\sqrt{56}$ (d) $\sqrt{72}$