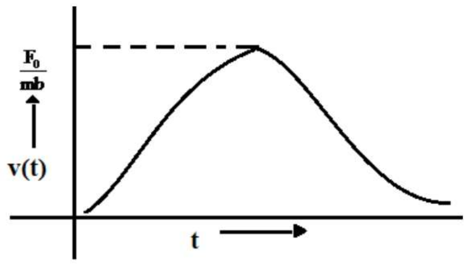
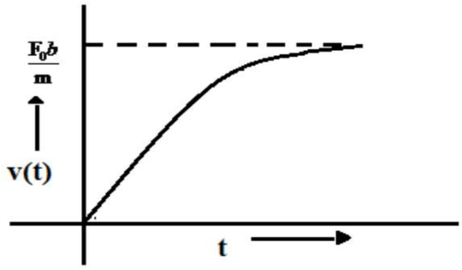
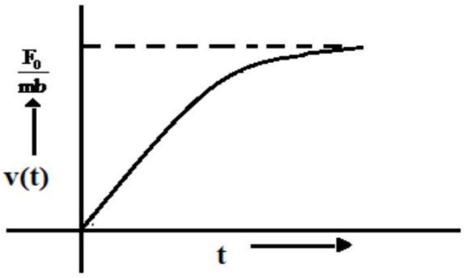
Questions With Solution

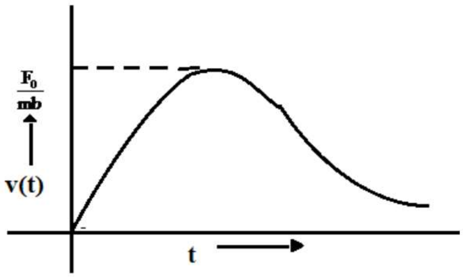
Section - A Physics

1. A particle of mass m is at rest at the origin at time t = 0. It is subjected to a force $F(t) = F_0e^{-bt}$ in the *x* direction. Its speed v(t) is depicted by which of the following curves?

(A) 

(B) 

(C) 

(D) 

Sol.

$F = F_0e^{-bt}$

$\Rightarrow \quad a = \frac{F}{m} = \frac{F_0}{m}e^{-bt}$

$\Rightarrow \quad \frac{dv}{dt} = \frac{F_0}{m}e^{-bt}$

$\int dv = \int_0^t \frac{F}{m}e^{-bt}dt$

$\Rightarrow \quad v = \frac{F}{m}\left[\frac{-1}{b}\right]\left[e^{-bt}\right]_0^t$

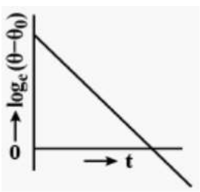
$\Rightarrow \quad v = -\frac{F}{mb}\left[e^{-bt} - 1\right]$

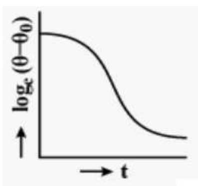
$v = 0$ at $t = 0$

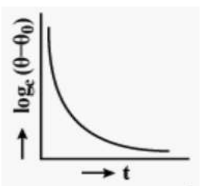
and $\quad v \to \frac{F}{mb}$ as $t \to \infty$

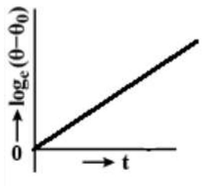
So, velocity increases continuously and attains a maximum value of $v = \frac{F}{mb}$ as $t \to \infty$.

2. A liquid in a beaker has temperature $\theta(t)$ at time t and θ_0 is temperature of surroundings, then according to Newton's law of cooling the correct graph between loge $(\theta - \theta_0)$ and t is

(A) 

(B) 

(C) 

(D) 

Sol. According to Newtons law of cooling.

$$\frac{d\theta}{dt} \propto -(\theta - \theta_0)$$

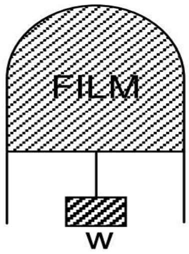
$$\Rightarrow \quad \frac{d\theta}{dt} = -k(\theta - \theta_0)$$

$$\int \frac{d\theta}{\theta - \theta_0} = \int -k\,dt$$

$\Rightarrow \quad \ln(\theta - \theta_0) = -kt + c$

Hence the plot of $\ln(\theta - \theta_0)$ vs t should be a straight line with negative slope.

3. A thin liquid film formed between a U-shaped wire and a light slider supports a weight of 1.5×10^{-2}N (see figure). The length of the slider is 30 cm and its weight negligible. The surface tension of the liquid film is



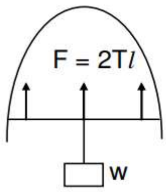
(1) 0.0125 Nm^{-1} (2) 0.1 Nm^{-1}
(3) 0.05 Nm^{-1} **(4)** 0.025 Nm^{-1}

Sol. The force of surface tension acting on the slider balances the force due to the weight.

$\Rightarrow \quad F = 2T\ell = w$

$\Rightarrow \quad 2T(0.3) = 1.5 \times 10^{-2}$

$\Rightarrow \quad T = 2.5 \times 10^{-2}$ N/m



4. A radar has a power of 1 Kw and is operating at a frequency of 10 GHz. It is located on a mountain top of height 500 m. The maximum distance upto which it can detect object located on the surface of the earth (Radius of earth = 6.4×10^6 m) is
(A) 80 km (B) 16 km (C) 40 km (D) 64 km

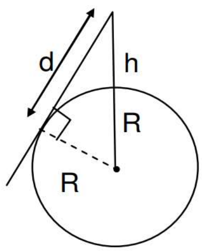
Sol. Maximum distance on earth where object can be detected is d, then

$$(h+R)^2 = d^2 + R^2$$

$\Rightarrow \quad d^2 = h^2 + 2Rh$

since $h << R$, $\Rightarrow \quad d^2 = 2hR$

$\Rightarrow \quad d = \sqrt{2(500)(6.4\times10^6)} = 80$ km



5. Two cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2, respectively. Their speeds are such that they make complete circles in the same time t. The ratio of their centripetal acceleration is
(A) $m_1r_1 : m_2r_2$ (B) $m_1 : m_2$ **(C)** $r_1 : r_2$ (D) 1 : 1

Sol. $a \propto r$

6. A guitar string is 90 cm long and has a fundamental frequency of 124 Hz. Where should it be pressed to produce a fundamental frequency of 180 Hz?
(A) 60 cm (B) 82 cm **(C)** 62 cm (D) 80 cm

Sol. The fundamental frequency of a string fixed at both ends is given by

$$\nu = \frac{1}{2L}\sqrt{\frac{F}{\mu}}.$$

As F and μ are fixed, $\frac{\nu_1}{\nu_2} = \frac{L_2}{L_1}$

or, $$L_2 = \frac{\nu_1}{\nu_2}L_1 = \frac{124\text{ Hz}}{180\text{ Hz}}(90\text{ cm}) = 62\text{ cm}.$$

Thus, the string should be pressed at 62 cm from an end.

7. This question has statement 1 and statement 2. Of the four choices given after the statements, choose the one that best describes the two statements.
If two springs S_1 and S_2 of force constants k_1 and k_2, respectively, are stretched by the same force, it is found that more work is done on spring S_1 than on spring S_2.
Statement 1 : If stretched by the same amount, work done on S_1, will be more than that on S_2
Statement 2 : $k_1 < k_2$
(A) Statement 1 is false, Statement 2 is true
(B) Statement 1 is true, Statement 2 is false
(C) Statement 1 is true, Statement 2 is the correct explanation for statement 1
(D) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation for statement 1.

Sol. Ans - A

$F = K_1S_1 = K_2 S_2$
$W_1 = FS_1, W_2 = FS_2$
$K_1S_1^2 > K_2S_2^2$
$S_1 > S_2$
$K_1 < K_2$
$W \propto K$
$W_1 < W_2$

8. A solid sphere of radius r made of a soft material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area a floats on the surface of the liquid, covering entire cross section of cylindrical container. When a mass m is placed on the surface of the piston to compress the liquid, the fractional decrement in the radius of the sphere, $\frac{dr}{r}$, is

(A) $\frac{mg}{Ka}$ (B) $\frac{Ka}{mg}$ (C) $\frac{Ka}{3mg}$ **(D)** $\frac{mg}{3Ka}$

Sol. $\frac{\Delta V}{V} = \frac{3\Delta r}{r}$

$K = \frac{P}{\Delta V / V}$

$\Rightarrow \frac{\Delta r}{r} = \frac{mg}{3Ka}$

9. In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision, is :

(A) $\frac{v_0}{\sqrt{2}}$ (B) $\frac{v_0}{4}$ **(C)** $\sqrt{2}\, v_0$ (D) $\frac{v_0}{2}$

Sol. $k_f = 1.5\, k_i$

$v_1^2 + v_2^2 = 1.5 v_0^2$

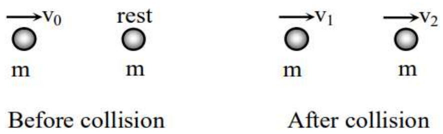
From conservation of momentum

$v_1 + v_2 = v_0$

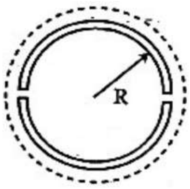
from (i) and (ii)

$2v_1v_2 = -0.5\; v_0^2$

So, $v_2 - v_1 = \sqrt{v_2^2 + v_1^2 - 2v_1v_2} = \sqrt{1.5v_0^2 + 0.5v_0^2} = \sqrt{2}\; v_0$



10. A wooden wheel of radius R is made of two semicircular parts (see figure); The two parts are held together by a ring made of a metal strip of cross sectional area S and length L. L is slightly less than $2\pi R$. To fit the ring on the wheel, it is heated so that its temperature rises by ΔT and it just steps over the wheel. As it cools down to surrounding temperature, it presses the semicircular parts together. If the coefficient of linear expansion of the metal is α and its Youngs'modulus is Y, the force that one part of the wheel applies on the other part is :



(A) $2\pi SY\alpha\Delta T$ (B) $SY\alpha\Delta T$ (C) $\pi SY\alpha\Delta T$ **(D)** $2SY\alpha\Delta T$

11. An aeroplane is going towards east at a speed of 540 km h^{-1} at a height of 1440 m. At a certain instant, the sound of the plane heard by a ground observer appears to come from a point vertically above him. Where is the plane at this instant? Speed of sound in air = 340ms^{-1}

(A) 825 m (B) 835 m **(C)** 600 m (D) 580 m

12. The internal energy of a monatomic ideal gas is 1.5 nRT. One mole of helium is kept in a cylinder of cross section 8.5 cm. The cylinder is closed by a light frictionless piston. The gas is heated slowly in a process during which a total of 42 J heat is given to the gas. If the temperature rises through 2°C, find the distance moved by the piston. Atmospheric pressure = 100 kPa.

(A) 0.2 cm **(B)** 20 cm (C) 0.4 cm (D) 40 cm

Sol. The change in internal energy of the gas is

$$\Delta U = 1{\cdot}5\, nR\,(\Delta T)$$
$$= 1{\cdot}5\,(1\text{ mol})\,(8{\cdot}3\text{ J K}^{-1}\text{ mol}^{-1})\,(2\text{ K})$$
$$= 24{\cdot}9\text{ J.}$$

The heat given to the gas = 42 J.

The work done by the gas is

$$\Delta W = \Delta Q - \Delta U$$
$$= 42\text{ J} - 24{\cdot}9\text{ J} = 17{\cdot}1\text{ J.}$$

If the distance moved by the piston is x, the work done is

$$\Delta W = (100\text{ kPa})\,(8{\cdot}5\text{ cm}^2)\,x.$$

Thus,

$$(10^5\text{ N m}^{-2})\,(8{\cdot}5\times10^{-4}\text{ m}^{-2})\,x = 17{\cdot}1\text{ J}$$

or, $x = 0{\cdot}2\text{ m} = 20\text{ cm.}$

13. If 'M' is the mass of water that rises in a capillary tube of radius 'r', then mass of water which will rise in a capillary tube of radius '2r' is:

(A) 4 M (B) $\frac{M}{2}$ (C) M (D) 2 M

Sol. Height of liquid rise in capillary tube $h = \frac{2T\cos\theta_C}{\rho r g}$

$\Rightarrow\ h \propto \frac{1}{r}$

When radius becomes double height become half

$\therefore\ h' = \frac{h}{2}$

Now, $M = \pi r^2 h \times \rho$ and $M' = \pi(2r)^2\,(h/2) \times \rho = 2M$.

14. One mole of an ideal monatomic gas is kept in a rigid vessel. The vessel is kept inside a steam chamber whose tempreature is 97°C. Initially, the temperature of the gas is 5.0°C. The walls of the vessel have an inner surface of area 800cm^2 and thickness 1.0 cm. If the temperature of the gas increases to 9.0°C in 1.0 seconds, find the thermal conductivity k ($Js^{-1}\ m^{-1}\ K^{-1}$) of the material of the walls. [$R = 8JK^{-1}mol^{-1}$]

(A) $K = \frac{1}{15}$ (B) $K = \frac{1}{16}$ (C) $K = \frac{2}{15}$ (D) $K = \frac{1}{8}$

Sol. The initial temperature difference is $97°C - 5°C = 92°C$ and at 5·0 s the temperature difference becomes $97°C - 9°C = 88°C$. As the change in the temperature difference is small, we work with the average temperature difference

$$\frac{92°C + 88°C}{2} = 90°C = 90\text{ K}.$$

The rise in the temperature of the gas is

$$9{\cdot}0°C - 5{\cdot}0°C = 4°C = 4\text{ K}.$$

The heat supplied to the gas in 5·0 s is

$$\Delta Q = nC_v\,\Delta T$$

$$= (1\text{ mol}) \times \left(\frac{3}{2} \times 8\text{ JK}^{-1}\text{ mol}^{-1}\right) \times (4\text{ K})$$

$$= 48\text{ J}.$$

If the thermal conductivity is K,

$$48\text{ J} = \frac{K(800 \times 10^{-4}\text{ m}^2) \times (90\text{ K})}{1{\cdot}0 \times 10^{-2}\text{ m}} \times 1{\cdot}0\text{ s}$$

or, $$K = \frac{48\text{ J}}{720\text{ msK}} = \frac{1}{15}\text{ J s}^{-1}\text{ m}^{-1}\text{ K}^{-1}.$$

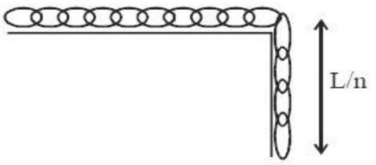
15. A uniform cable of mass 'M' and length 'L' is placed on a horizontal surface such that its $\left(\frac{1}{n}\right)^{th}$ part is hanging below the edge of the surface. To lift the hanging part of the cable upto the surface, the work done should be:

(A) nMgL **(B)** $\frac{MgL}{2n^2}$ (C) $\frac{2MgL}{n^2}$ (D) $\frac{MgL}{n^2}$

Sol. Mass of the hanging part = $\frac{M}{n}$

$h_{COM} = \frac{L}{2n}$

Work done $W = mgh_{COM} = \left(\frac{M}{n}\right)g\left(\frac{L}{2n}\right) = \frac{MgL}{2n^2}$



16. A source emitting sound of frequency 180 Hz is placed in front of a wall at a distance of 2 m from it. A detector is also placed in front of the wall at the same distance from it. Find the minimum distance between the source and the detector for which the detector detects a maximum of sound. Speed of sound in air = 360 ms^{-1}.

(A) 4 m (B) 2.5 m **(C)** 3 m (D) 5 m

Sol.

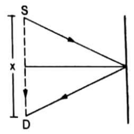


Figure 16-W3

The situation is shown in figure (16-W3). Suppose the detector is placed at a distance of x meter from the source. The direct wave received from the source travels a distance of x meter. The wave reaching the detector after reflection from the wall has travelled a distance of $2[(2)^2 + x^2/4]^{1/2}$ metre. The path difference between the two waves is

$$\Delta = \left\{ 2\left[(2)^2 + \frac{x^2}{4}\right]^{1/2} - x \right\} \text{ metre.}$$

Constructive interference will take place when $\Delta = \lambda, 2\lambda, \ldots$. The minimum distance x for a maximum corresponds to

$$\Delta = \lambda. \qquad \ldots \text{(i)}$$

The wavelength is $\lambda = \frac{v}{\nu} = \frac{360 \text{ m s}^{-1}}{180 \text{ s}^{-1}} = 2$ m.

Thus, by (i), $2\left[(2)^2 + \frac{x^2}{4}\right]^{1/2} - x = 2$

or, $\left[4 + \frac{x^2}{4}\right]^{1/2} = 1 + \frac{x}{2}$

or, $4 + \frac{x^2}{4} = 1 + \frac{x^2}{4} + x$

or, $3 = x$.

Thus, the detector should be placed at a distance of 3 m from the source. Note that there is no abrupt phase change.

17. A diatomic gas (γ = 1.4) does 200 J of work when it is expanded isobarically. Find the heat given to the gas in the process.
(A) 600 J (B) 500 J **(C)** 700 J (D) - 200 J

Sol. **For a diatomic gas, $C_V = \frac{5}{2}R$ and $C_p = \frac{7}{2}R$. The work done in an isobaric process is**

$$W = p(V_2 - V_1)$$
$$= nRT_2 - nRT_1$$

or, $T_2 - T_1 = \frac{W}{nR}$.

The heat given in an isobaric process is

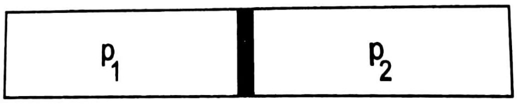
$$Q = nC_p(T_2 - T_1)$$
$$= nC_p \frac{W}{nR} = \frac{7}{2}W$$
$$= \frac{7}{2} \times 200 \text{ J} = 700 \text{ J}.$$

18. A person going away from a factory on his scooter at a speed of 36 kmh listens to the siren of the factory. If the main frequency of the siren is 700 Hz and a wind is blowing along the direction of the scooter at 36 km h^{-1}, find the main frequency as heard by the person
(A) 710 Hz **(B)** 680 Hz (C) 715 Hz (D) 675 Hz

Sol. The speed of sound in still air is 340 ms^{-1}. Let us work from the frame of reference of the air. As both the observer and the wind are moving at the same speed along the same direction with respect to the ground, the observer is at rest with respect to the medium. The source (the siren) is moving with respect to the wind at a speed of 36 km h^{-1}, i.e., 10 m s^{-1}. As the source is going away from the observer who is at rest with respect to the medium, the frequency heard is

$$v' = \frac{v}{v+u_s}v = \frac{340}{340+10} \times 700Hz = 680$$

19. Figure shows a cylindrical tube of volume V_0 divided in two parts by a frictionless separator. The walls of the tube are adiabatic but the separator is conducting. Ideal gases are filled in the two parts. When the separator is kept in the middle, the pressures are p_1 and p_2 in the left part and the right part respectively. The separator is slowly slid and is released at a position where it can stay in equilibrium. Find the volume of the left part.



(A) $\frac{p_2 v_0}{p_1 + p_2}$ **(B)** $\frac{p_1 v_0}{p_1 + p_2}$ (C) $\frac{(p_1 + p_2)v_0}{p_1}$ (D) $\frac{(p_1 + p_2)v_0}{p_2}$

Sol. As the separator is conducting, the temperatures in the two parts will be the same. Suppose the common temperature is T when the separator is in the middle. Let n_1 and n_2 be the number of moles of the gas in the left part and the right part respectively. Using ideal gas equation,

$$p_1 \frac{V_0}{2} = n_1 RT$$

and $$p_2 \frac{V_0}{2} = n_2 RT.$$

Thus, $$\frac{n_1}{n_2} = \frac{p_1}{p_2}. \quad \ldots \text{(i)}$$

The separator will stay in equilibrium at a position where the pressures on the two sides are equal. Suppose the volume of the left part is V_1 and of the right part is V_2 in this situation. Let the common pressure be p'. Also, let the common temperature in this situation be T'. Using ideal gas equation,

$$p'V_1 = n_1 RT'$$

and $$p'V_2 = n_2 RT'$$

or, $$\frac{V_1}{V_2} = \frac{n_1}{n_2} = \frac{p_1}{p_2}. \quad \text{[using (i)]}$$

Also, $$V_1 + V_2 = V_0.$$

Thus, $$V_1 = \frac{p_1 V_0}{p_1 + p_2} \text{ and } V_2 = \frac{p_2 V_0}{p_1 + p_2}.$$

20. A boy can throw a stone up to a maximum height of 10 m. The maximum horizontal distance that the boy can throw the same stone up to will be

(A) $20\sqrt{2}m$ (B) 10 m (C) $10\sqrt{2}m$ **(D)** 20 m

Sol. maximum vertical height = $\frac{u^2}{2g} = 10\,m$

Horizontal range of a projectile = $\frac{u^2 \sin 2\theta}{g}$

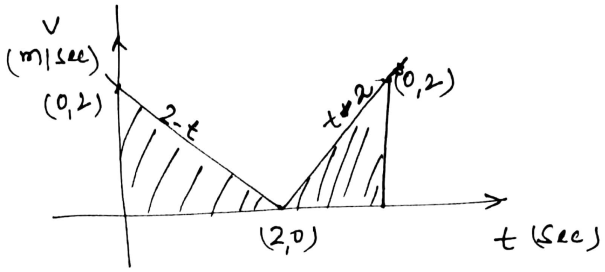
Range is maximum when $\theta = 45^0$

Maximum horizontal range = $\frac{u^2}{g}$

Hence maximum horizontal distance = 20 m.

1. A bird flies for 4 s with a velocity of $|t - 2|$ m/s in a straight line, where t = time in seconds. It covers a distance of ____m.

Sol.



Area of Shedded Region :- $\frac{1}{2}\times 2\times 2+\frac{1}{2}\times 2\times 2 = 4m.$

2. The constant γ for oxygen as well as for hydrogen is 1.40. If the speed of sound in oxygen is 470 ms^{-1}, the speed in hydrogen at the same temperature and pressure will be ____m/s.

Sol. The speed of sound in a gas is given by $v=\sqrt{\frac{\gamma P}{\rho}}$. At STP, 22·4 litres of oxygen has a mass of 32 g whereas the same volume of hydrogen has a mass of 2 g. Thus, the density of oxygen is 16 times the density of hydrogen at the same temperature and pressure. As γ is same for both the gases,

$$\frac{v\text{ (hydrogen)}}{v\text{ (oxygen)}}=\sqrt{\frac{\rho\text{(oxygen)}}{\rho\text{ (hydrogen)}}}$$

or, v(hydrogen) $= 4\,v$ (oxygen)

$= 4 \times 470\ \text{m s}^{-1} = 1880\ \text{m s}^{-1}$.

3. A lead bullet penetrates into a solid object and melts. The initial temperature of the bullet is 27°C and its melting point is 327°C. Latent heat of fusion of lead = 2.5 × 10 J kg^{-1} and specific heat capacity of lead = 125 J kg^{1} K^{1}. Assuming that 50% of its kinetic energy was used to heat it, the initial speed of the bullet is ____m/s.

Sol. Let the mass of the bullet = m.

Heat required to take the bullet from 27°C to 327°C

$$= m \times (125\ \text{J kg}^{-1}\ \text{K}^{-1})\ (300\ \text{K})$$

$$= m \times (3{\cdot}75 \times 10^{4}\ \text{J kg}^{-1}).$$

Heat required to melt the bullet

$$= m \times (2{\cdot}5 \times 10^{4}\ \text{J kg}^{-1}).$$

If the initial speed be v, the kinetic energy is $\frac{1}{2}mv^{2}$ and hence the heat developed is $\frac{1}{2}\left(\frac{1}{2}mv^{2}\right)=\frac{1}{4}mv^{2}$. Thus,

$$\frac{1}{4}mv^{2} = m(3{\cdot}75 + 2{\cdot}5) \times 10^{4}\ \text{J kg}^{-1}$$

or, $v = 500\ \text{m s}^{-1}$.

4. A tuning fork of frequency 480 Hz is used in an experiment for measuring speed of sound (v) in air by resonance tube method. Resonance is observed to occur at two successive lengths of the air column, $\ell_1 = 30$ cm and $\ell_2 = 70$ cm. Then v is equal to ____m/s.

Sol. $v = 2f(l_2 - l_1)$

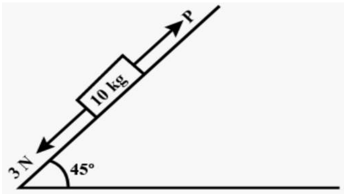
$v = 2 \times 480 \times (70 - 30) \times 10^{-2}$

$v = 960 \times 40 \times 10^{-2}$

$v = 38400 \times 10^{-2}$ m/s

$v = 384$ m/s

5. A block of mass 10 kg is kept on a rough inclined plane as shown in the figure. A force of 3 N is applied on the block. The coefficient of static friction between the plane and the block is 0.6. The minimum value of force P, such that the block does not move downward will be _____N. (take $g = 10\ ms^{-2}$)



Sol. For equilibrium of the block net force should be zero. Hence we can write.

$mg\sin\theta + 3 = P + \text{friction}$

$mg\sin\theta + 3 = P + \mu mg\cos\theta$.

After solving, we get, P = 32 N.

6. The sum of two forces $\vec{P}$ and $\vec{Q}$ is $\vec{R}$ such that $|\vec{R}| = |\vec{P}|$. The angle θ (in degrees) that the resultant of $2\vec{P}$ and $\vec{Q}$ will make with $\vec{Q}$ is, _____°.

Sol. $|\vec{P} + \vec{Q}| = |\vec{P}|$

$P^2 + Q^2 + 2PQ\cos\theta = P^2$

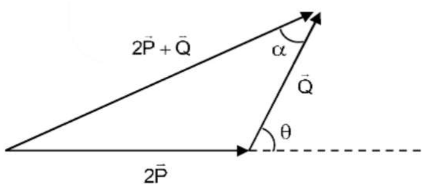
$\Rightarrow \quad Q + 2P\cos\theta = 0$

$\Rightarrow \quad \cos\theta = -\dfrac{Q}{2P}$

$\tan\alpha = \dfrac{2P\sin\theta}{2P\cos\theta + Q} = \infty$

$\because \quad [2P\cos\theta + Q = 0]$

$\alpha = 90°$



7. Two travelling waves produces a standing wave represented by equation, $y = 1.0\text{ mm}\cos(1.57\text{cm}^{-1}) \times \sin(78.5\text{s}^{-1})t$. The node closest to the origin in the region $x > 0$ will be at x = _____cm.

Sol. $kx = \pi/2$

$\Rightarrow 1.57x = \dfrac{\pi}{2} = \dfrac{3.14}{2} = 1.57$

$x = 1\text{cm}$

8. Three containers C_1, C_2 and C_3 have water at different temperatures. The table below shows the final temperature T when different amounts of water (given in liters) are taken from each container and mixed (assume no loss of heat during the process)

C_1	C_2	C_3	T
1ℓ	2ℓ	__	60ºC
__	1ℓ	2ℓ	30ºC
2ℓ	__	1ℓ	60ºC
1ℓ	1ℓ	1ℓ	θ

The value of θ (in °C to the nearest integer) is _________.

Sol. $1\theta_1 + 2\theta_2 = (1 + 2)\,60$

$\theta_1 + 2\theta_2 = 180$ ...(1)

$0 \times \theta_1 + 1 \times \theta_2 + 2 \times \theta_3 = (1 + 2)\,30$

$\theta_2 + 2\theta_3 = 90$ ...(2)

$2 \times \theta_1 + 0 \times \theta_2 + 1 \times \theta_3 = (2 + 1)\,60$

$2\theta_1 + \theta_3 = 180$ ...(3)

and $\theta_1 + \theta_2 + \theta_3 = (1 + 1 + 1)\,\theta$ ...(4)

from (1) + (2) + (3)

$3\theta_1 + 3\theta_2 + 3\theta_3 = 450 \Rightarrow \theta_1 + \theta_2 + \theta_3 = 150$

From (4) equation $150 = 3\theta \Rightarrow \theta = 50°C$

9. A heat engine operates between a cold reservoir at temperature $T_2 = 300K$ and a hot reservoir at temperature T_1. It takes 200 J of heat from the hot reservoir and delivers 120 J of heat to the cold reservoir in a cycle. The minimum temperature of the hot reservoir will be ____K.

Sol. The work done by the engine in a cycle is

$$W = 200\text{ J} - 120\text{ J} = 80\text{ J}.$$

The efficiency of the engine is

$$\eta = \frac{W}{Q} = \frac{80\text{ J}}{200\text{ J}} = 0{\cdot}40.$$

From Carnot's theorem, no engine can have an efficiency greater than that of a Carnot engine.

Thus, $0{\cdot}40 \le 1 - \frac{T_2}{T_1} = 1 - \frac{300\text{ K}}{T_1}$

or, $\frac{300\text{ K}}{T_1} \le 1 - 0{\cdot}40 = 0{\cdot}60$

or, $T_1 \ge \frac{300\text{ K}}{0{\cdot}60}$

or, $T_1 \ge 500\text{ K}.$

The minimum temperature of the hot reservoir has to be 500 K.

10. Two simple harmonic motions are represented by the equations $x_1 = 5\sin\left(2\pi t + \frac{\pi}{4}\right)$ and $x_2 = 5\sqrt{2}\sin(2\pi t + \cos 2\pi t)$. The amplitude of second motion is ________ times the amplitude in first motion.

Sol. $x_1 = 5\sin\left(2\pi t + \frac{\pi}{4}\right)$

$x_2 = 5\sqrt{2}(\sin 2\pi t + \cos 2\pi t)$

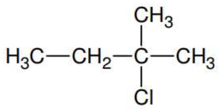
$= 10\sin\left(2\pi t + \frac{\pi}{4}\right)$

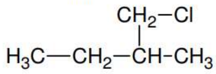
$x_{2,max} = 2.x_{1,max}$

Chemistry Section - A
With Solution

1. How many chiral compounds are possible on monochlorination of 2–methyl butane ?
(a) 8 **(b)** 2 (c) 4 (d) 6

Sol. $H_3C-CH_2-CH(CH_3)-CH_3$ on monochlorination gives
$H_2C(Cl)-CH_2-CH(CH_3)-CH_3$ (I) Achiral
$H_3C-CH(Cl)-CH(CH_3)-CH_3$ (II) Chiral


(III) Achiral


(IV) chiral

2. For gaseous state, if most probable speed is denoted by C*, average speed by $\overline{C}$ and mean square speed by C, then for a large number of molecules the ratios of these speeds are:
(A) $C^* : \overline{C} : C = 1.128 : 1.225 : 1$
(B) $C^* : \overline{C} : C = 1 : 1.128 : 1.225$
(C) $C^* : \overline{C} : C = 1 : 1.125 : 1.128$
(D) $C^* : \overline{C} : C = 1.225 : 1.128 : 1$

Sol. $C^* = \sqrt{\frac{2RT}{M}}$, $\overline{C} = \sqrt{\frac{8RT}{\pi M}}$, $C = \sqrt{\frac{3RT}{M}}$

3. Which of the following on thermal decomposition yields a basic as well as an acidic oxide ?
(A) $NaNO_3$ (B) $KClO_3$ **(C)** $CaCO_3$ (D) NH_4NO_3

Sol. $CaCO_3 \rightarrow \underset{Basic}{CaO} + \underset{Acidic}{CO_2}$

4. The species which can best serve as an initiator for the cationic polymerization is :
(A) $LiAlH_4$ (B) HNO_3 **(C)** $AlCl_3$ (D) BuLi

Sol. lewis acids can initiate the cationic polymerization.

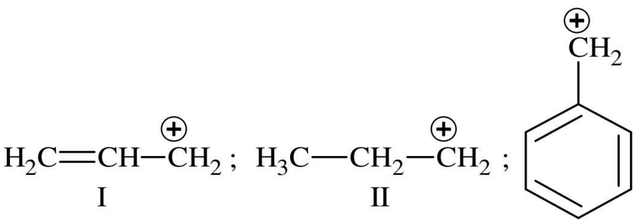
5. The molecule having smallest bond angle is :
(A) NCl_3 (B) $AsCl_3$ **(C)** $SbCl_3$ (D) PCl_3

Sol. As the size of central atom increases lone pair bond pair repulsions increases so, bond angle decreases

6. The standard reduction potentials for Zn^{2+}/Zn, Ni^{2+}/Ni, and Fe^{2+}/Fe are – 0.76, – 0.23 and – 0.44 V respectively. The reaction $X + Y^{2+} \rightarrow X^{2+} + Y$ will be spontaneous when :
(A) X = Ni, Y = Fe (B) X = Ni, Y = Zn (C) X = Fe, Y = Zn **(D)** X = Zn, Y = Ni

Sol. $Zn + Fe^{+2} \rightarrow Zn^{+2} + Fe$
$Fe + Ni^{+2} \rightarrow Fe^{2+} + Ni$
$Zn + Ni^{2+} \rightarrow Zn^{+2} + Ni$
All these are spontaneous

7. The order of stability of the following carbocations:


I II III is :

(A) II > III > I (B) I > II > III **(C)** III > I > II (D) III > II > I

Sol. Order of stability is III > I > II
(Stability $\propto$ extent of delocalization)

8. The ratio of masses of oxygen and nitrogen in a particular gaseous mixture is 1 : 4. The ratio of number of their molecule is:
(A) 1: 8 (B) 3 : 16 (C) 1 : 4 **(D)** 7 : 32

Sol. $\text{Moles of } O_2 = \frac{w}{32}$

$\text{Moles of } N_2 = \frac{4w}{28}$

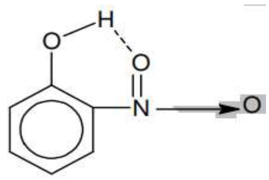
$$\frac{n_{O_2}}{n_{N_2}} = \frac{w}{32} \times \frac{28}{4w} = \frac{7}{32}$$

9. Experimentally it was found that a metal oxide has formula $M_{0.98}O$. Metal M, is present as M^{2+} and M^{3+} in its oxide. Fraction of the metal which exists as M^{3+} would be:
(A) 4.08% (B) 6.05% (C) 5.08% (D) 7.01%

Sol. Metal oxide = $M_{0.98}O$
If 'x' ions of M are in +3 state, then
$3x + (0.98 - x) \times 2 = 2$
$x = 0.04$

So the percentage of metal in +3 state would be $\frac{0.04}{0.98} \times 100 = 4.08\%$

10. Ortho–Nitrophenol is less soluble in water than p– and m– Nitrophenols because :
(1) o–Nitrophenol is more volatile in steam than those of m – and p–isomers
(2) o–Nitrophenol shows Intramolecular H–bonding
(3) o–Nitrophenol shows Intermolecular H–bonding
(4) Melting point of o–Nitrophenol is lower than those of m–and p–isomers.

Sol. 

Intramolecular H–bonding decreases water solubility.

11. The equilibrium constant (KC) for the reaction $N_2(g) + O_2(g) \rightarrow 2NO(g)$ at temperature T is 4×10^{-4}. The value of KC for the reaction, $NO(g) \rightarrow \frac{1}{2} N_2(g) + \frac{1}{2} O_2(g)$ at the same temperature is :
(A) 0.02 (B) 2.5×10^2 (C) 4×10^{-4} **(D)** 50.0

Sol. $N_2 + O_2 \rightleftharpoons 2NO \qquad K_C = 4 \times 10^{-4}$

$NO \rightleftharpoons \frac{1}{2}N_2 + \frac{1}{2}O_2 \qquad K_C^1 = \sqrt{\frac{1}{K_C}}$

$$K_C^1 = \frac{1}{\sqrt{4 \times 10^{-4}}} = 50$$

12. How many litres of water must be added to 1 litre of an aqueous solution of HCl with a pH of 1 to create an aqueous solution with pH of 2?
(A) 0.9 L (B) 2.0 L **(C)** 9.0 L (D) 0.1 L

Sol. Initial pH = 1, i.e. $[H^+] = 0.1$ mole/litre
New pH = 2, i.e. $[H^+] = 0.01$ mole/litre
In case of dilution: $M_1V_1 = M_2V_2$
$0.1 \times 1 = 0.01 \times V_2$
$V_2 = 10$ litre.
Volume of water added = 9 litre.

13. Stability of the species Li_2, Li_2^- and Li_2^+ increases in the order of:

(A) $Li_2^- < Li_2^+ < Li_2$ (B) $Li_2 < Li_2^- < Li_2^+$ (C) $Li_2^- < Li_2 < Li_2^+$ (D) $Li_2 < Li_2^+ < Li_2^-$

Sol. $Li_2(6) = \sigma 1s^2\ \overset{*}{\sigma}1s^2\ \sigma 2s^2$

$$B.O. = \frac{4-2}{2} = 1$$

$Li_2^+(5) = \sigma 1s^2\ \overset{*}{\sigma}1s^2 \sigma 2s^1$

$$B.O. = \frac{3-2}{2} = 0.5$$

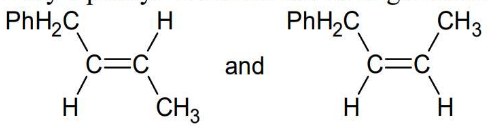
$Li_2^-(7) = \sigma 1s^2\ \overset{*}{\sigma}1s^2 \sigma 2s^2\ \overset{*}{\sigma}2s^1$

$$B.O. = \frac{4-3}{2} = 0.5$$

Li_2^+ is more stable than Li_2^- because Li_2^- has more numbers of antibonding electrons.

14. Which of the following compounds will exhibit geometrical isomerism?
(A) 3-Phenyl-1-butene (B) 2-Phenyl-1-butene
(C) 1,1-Diphenyl-1-propane (D) 1-Phenyl-2-butene

Sol. Only 1 phenyl -2-butene can show geometrical isomerism.



trans cis

15. 3 g of activated charcoal was added to 50 mL of acetic acid solution (0.06N) in a flask. After an hour it was filtered and the strength of the filtrate was found to be 0.042 N. The amount of acetic acid adsorbed (per gram of charcoal) is:
(A) 36 mg (B) 42 mg **(C)** 54 mg (D) 18 mg

Sol. Amount adsorbed
$= (0.060 - 0.042) \times 50 \times 10^{-3} \times 60$
$= 0.018 \times 50 \times 60 \times 10^{-3}$
$= .018 \times 3$
$= .054$ gm $= 54$ mg

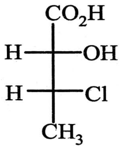
Amount adsorbed per gram of activated charcoal $= \frac{54}{3} = 18$ mg

16. The correct set of four quantum numbers for the valence elections of rubidium atom (Z= 37) is:

(A) $5, 1, 1, +\frac{1}{2}$ (B) $5, 0, 1, +\frac{1}{2}$ **(C)** $5, 0, 0, +\frac{1}{2}$ (D) $5, 1, 0, +\frac{1}{2}$

Sol. $_{37}Rb = [Kr]\,5s^1$

$n = 5,\ l = 0,\ m = 0,\ s = +\frac{1}{2}$

17. The absolute configuration of  is :

(A) (2S, 3S) (B) (2R, 3R) (C) (2R, 3S) **(D)** (2S, 3R)

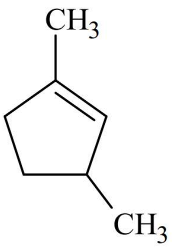
18. Considering the basic strength of amines in aqueous solution, which one has the smallest pK_b value?

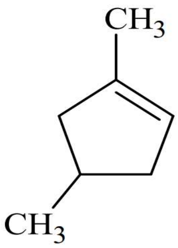
(A) $(CH_3)_3N$ (B) $C_6H_5NH_2$ **(C)** $(CH_3)_2NH$ (D) CH_3NH_2

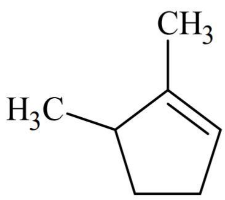
Sol. Aliphatic amines are more basic than aromatic amines.

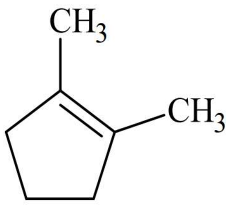
$(CH_3)_2NH > CH_3NH_2 > (CH_3)_3N$ (among aliphatic amines in water).

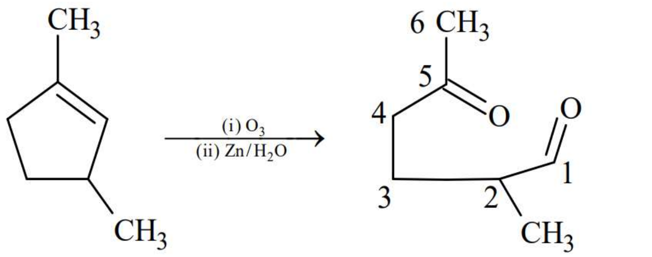
19. Which compound would give 5-keto-2-methyl hexanal upon ozonolysis?

(A) 

(B) 

(C) 

(D) 

Sol. 

5 - keto- 2 - methyl hexanal

20. If Z is a compressibility factor, van der Waals equation at low pressure can be written as:

(A) $Z = 1 - \frac{Pb}{RT}$ (B) $Z = 1 + \frac{Pb}{RT}$ (C) $Z = 1 + \frac{RT}{Pb}$ **(D)** $Z = 1 - \frac{a}{VRT}$

Sol. $\left(P+\frac{n^2a}{V^2}\right)(V-nb)=nRT$

For 1 mole, $\left(P+\frac{a}{V^2}\right)(V-b)=RT$

$$PV = RT + Pb - \frac{a}{V} + \frac{ab}{V^2}$$

at low pressure, terms Pb & $\frac{ab}{V^2}$ will be negligible as compared to RT.

So, $PV = RT - \frac{a}{V}$

$$Z = 1 - \frac{a}{RTV}$$

Section - B

1. How many possible optical isomerism in cyclic form of frucutose

ans :- 16

2. 6.023×10^{22} molecules are present in 10 g of a substance 'x'. The molarity of a solution containing 5 g of substance 'x' in 2 L solution is ___________ $\times 10^{-3}$

Sol. Number of mole of X = $\frac{6.022\times10^{22}}{6.022\times10^{23}} = \frac{10}{\text{Molar mass of X}}$

So molar mass of X = 100g

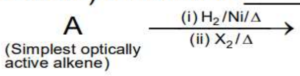
Molarity = $\frac{5}{100\times2} = 0.025M$

Ans. = 0.025 M

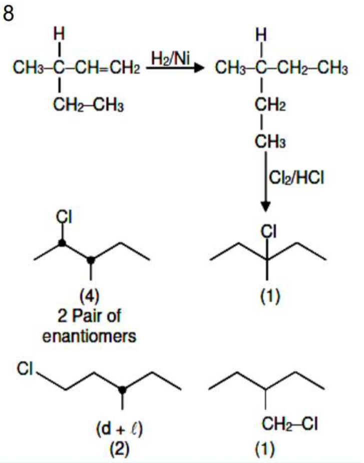
$M = 25\times10^{-3}$

So P = 25

3. The total number of monohalogenated organic products in the following (including stereoisomers) reaction is __________.



Sol. 8



4. The mole fraction of glucose ($C_6H_{12}O_6$) in an aqueous binary solution is 0.1. The mass percentage of water in it, to the nearest integer, is _________.

Sol. 47

Let total mole of solution = 1

So mole of glucose = 0.1

Mole of H_2O = 0.9

$$\%(w/w) \text{ of } H_2O = \left[\frac{0.9\times18}{0.9\times18+0.1\times180}\right]\times100 = 47.368 = 47.37$$

5. Total intensive properties

Boiling point, Entropy, pH, EMF of a cell, Volume, Surface tension

ans : - 4

6. The OH^- concentration in a mixture of 5.0 mL of 0.0504 M NH_4Cl and 2 mL of 0.0210 M NH_3 solution is $x \times 10^{-6}$ M. The value of x is _______. (Nearest integer)

[Given $K_w = 1 \times 10^{-14}$ and $K_b = 1.8 \times 10^{-5}$]

Sol. 0.0504 M NH_4Cl of 5ml $\Rightarrow$ millimole of $NH_4^+ = 0.0504\times5$

0.0210 M NH_3 of 2ml $\Rightarrow$ millimole of $NH_3 = 0.0210\times2$

It is a basic buffer.

Total volume = 7ml

$$\text{Here, } K_b = \frac{[OH^-]\times[NH_4^+]}{[NH_4OH]}$$

$$\therefore 1.8\times10^{-5} = \frac{[OH^-]\times0.0504\times5}{0.0210\times2}$$

$$\therefore [OH^-] = \frac{1.8\times10^{-5}\times0.0210\times2}{0.0504\times5} = 0.3\times10^{-5}M$$

$$\therefore [OH^-] = 3\times10^{-6}M$$

$\therefore x = 3$

Ans. = 3

7. The number of 4f electrons in the ground state electronic configuration of Gd^{2+} is _____.

[Atomic number of Gd = 64]

Sol. Electronic configuration of

Gd is ; $Gd_{64} = [Xe]4f^7 5d^1 6s^2$

Hence $Gd_{64}^{++} = [Xe]4f^7 5d^1 6s^0$

Total number of electrons in 4f sub-shell = 7

Ans. = 7

8. These are physical properties of an element

(A) Sublimation enthalpy

(B) Ionisation enthalpy

(C) Hydration enthalpy

(D) Electron gain enthalpy

The total number of above properties that affect the reduction potential is _________ (Integer answer)

Sol. Except electron gain enthalpy all the physical properties like sublimation enthalpy, ionization enthalpy and hydration energy affected the value of reduction potential.

Ans. =3

9. A chloro compound "A"

(i) forms aldehydes on ozonolysis followed by the hydrolysis.

(ii) when vaporized completely 1.53 g of A, gives 448 mL of vapour at STP.

The number of carbon atoms in a molecule of compound A is ________.

Sol.

$$\underbrace{A}_{\text{Chloro compound}} \xrightarrow[(2)\ H_2O]{(1)\ O_3} \text{Formaldehyde}$$

↓

Weight = 1.53 gm

V = 448 ml(STP)

$$\text{Mole} = \frac{448}{22400} = \frac{\text{weight}}{\text{molecular weight}}$$

$$\therefore \frac{1.53}{\text{molecular weight}} = \frac{1}{50}$$

$\therefore$ Molecular weight of compound A = $50 \times 1.53 = 76.5$ gram

Molecular weight = $76.5 = C_nH_{2n-1}Cl$

$\therefore C_nH_{2n-1} = 41$

or $12n + 2n - 1 = 41$

$$\therefore n = \frac{42}{14} = 3$$

Molecular formula = C_3H_5Cl

Number of C- atoms = 3

10. The equilibrium constant K_c at 298 K for the reaction

$A + B \rightleftharpoons C + D$

is 100. Starting with an equimolar solution with concentrations of A, B, C and D all equal to 1 M, the equilibrium concentration of D is ________ × 10^{-2} M. (Nearest integer)

Sol. For equation $A + B \rightleftharpoons C + D$

$$K_C = \frac{[C][D]}{[A][B]} = 100 \text{ at } 298K$$

Here;	A +	B $\rightleftharpoons$	C +	D
t = 0	1M	1M	1M	1M
eq.	(1-x)	(1-x)	(1+x)	(1+x)

$$Kc = \frac{(1+x)\times(1+x)}{(1-x)\times(1-x)}$$

$$\therefore 100 = \frac{(1+x)^2}{(1-x)^2}$$

$$\therefore \frac{1+x}{1-x} = 10$$

$$1 + x = 10 - 10x$$

$$\therefore x = \frac{9}{11}$$

At equilibrium, concentration of D is = $1 + \frac{9}{11} = \frac{11+9}{11} = \frac{20}{11}$

$= 1.818 = 181.8 \times 10^{-2}M$

Ans. = 182 (Nearest integer)

Questions With Solution

Section - A Mathematics

1. The locus of the point of intersection of the lines $\sqrt{3}x - y - 4\sqrt{3}k = 0$ and $\sqrt{3}kx + ky - 4\sqrt{3} = 0$ for different values of k is

(A) ellipse (B) parabola (C) circle **(D)** hyperbola

Sol. D

$\sqrt{3}\,x - y - 4\sqrt{3}\,k = 0$ ...(1)

$\sqrt{3}\,kx + ky - 4\sqrt{3} = 0$ ...(2)

Solve (1) and (2)

$x = 2\frac{(1+k^2)}{k}$ and $y = \frac{2\sqrt{3}(1-k^2)}{k}$

$\frac{x^2}{4} - \frac{y^2}{12} = 4 \Rightarrow \frac{x^2}{16} - \frac{y^2}{48} = 1$ Hyperbola

2. The ratio $\dfrac{2^{\log_{2^{1/4}} a} - 3^{\log_{27}(a^2+1)^3} - 2a}{7^{4\log_{49} a} - a - 1}$ simplifies to

(a) $a^2 - a - 1$ (b) $a^2 + a - 1$ (c) $a^2 - a + 1$ (4) $a^2 + a + 1$

Sol. **D**

$$= \frac{2^{\log_2(a^4)} - 3^{\log_3(a^2+1)} - 2a}{7^{\log_7(a^2)} - a - 1} = \frac{a^4 - (a^2+1) - 2a}{a^2 - a - 1}$$

$$= \frac{(a^2)^2 - (a+1)^2}{(a^2 - a - 1)} = a^2 + a + 1$$

3. The value of the expression $\left(1+\cos\frac{\pi}{10}\right)\left(1+\cos\frac{3\pi}{10}\right)\left(1+\cos\frac{7\pi}{10}\right)\left(1+\cos\frac{9\pi}{10}\right)$ is

(A) 1/8 (D) 1/16 (C) 1/4 (D) 0

Sol. **B**

$$\left(1+\cos\frac{\pi}{10}\right)\left(1+\cos\frac{3\pi}{10}\right)\left(1+\cos\frac{7\pi}{10}\right)\left(1+\cos\frac{9\pi}{10}\right)$$

$$= \left(1+\cos\frac{\pi}{10}\right)\left(1+\cos\frac{3\pi}{10}\right)\left(1-\cos\frac{3\pi}{10}\right)\left(1-\cos\frac{\pi}{10}\right)$$

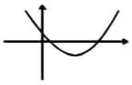
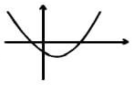
$$= \left(1-\cos^2\frac{\pi}{10}\right)\left(1-\cos^2\frac{3\pi}{10}\right) = \sin^2\frac{\pi}{10}\ \sin^2\frac{3\pi}{10}$$

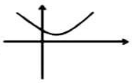
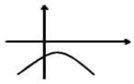
$$= \sin^2\left(\frac{\pi}{2}-\frac{4\pi}{10}\right)\ \sin^2\left(\frac{\pi}{2}-\frac{2\pi}{10}\right)$$

$$= \left[\cos\left(\frac{4\pi}{10}\right)\cos\left(\frac{2\pi}{10}\right)\right]^2 = \left(\cos\frac{2\pi}{5}.\cos\frac{\pi}{5}\right)^2$$

$$= \left(\frac{\sin\frac{4\pi}{5}}{2^2\sin\frac{\pi}{5}}\right)^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$$

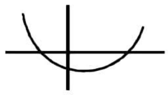
4. Which of the following graph represents expression 2 $f(x) = ax^2 + bx + c$ ($a \neq 0$) when $a > 0$, $b < 0$ & $c < 0$?

(A)  (B) 

(C)  (D) 

Sol. **B**

$f(x) = ax^2 + bx + c$

$\alpha + \beta = \frac{-b}{a} > 0,\ b^2 - 4ac > 0$ 

$\alpha\beta = \frac{c}{a} < 0$

5. The number of the integer solutions of $x^2 + 9 < (x+3)^2 < 8x + 25$ is

(A) 1 (B) 2 (C) 3 (D) None of these

Sol. **D**

$x^2 + 9 < (x+3)^2 < 8x + 25$

$x^2 + 9 < x^2 + 6x + 9 \Rightarrow x > 0$

& $(x+3)^2 < 8x + 25$

$x^2 + 6x + 9 - 8x - 25 < 0$

$x^2 - 2x - 16 < 0$

$1 - \sqrt{17} < x < 1 + \sqrt{17}$ & $x > 0$

$\Rightarrow x \in (0, 1 + \sqrt{17})$

Integer x = 1, 2, 3, 4, 5

No. of integer are = 5

6. If $\frac{1}{1^2} + \frac{1}{1^2} + \frac{1}{3^2} + \text{.......upto} \propto = \frac{\pi^2}{6}$, then $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ... =$

(A) $\frac{\pi^2}{12}$ (B) $\frac{\pi^2}{24}$ (C) $\frac{\pi^2}{8}$ (D) None of these

Sol. **c**

$S = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + ... \infty = \frac{\pi^2}{6}$

Now $S_{even} = \frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + ... \infty$

$= \frac{1}{2^2}\left[1 + \frac{1}{2^2} + \frac{1}{3^2} + ...\infty\right] = \frac{1}{2^2}\ \frac{\pi^2}{6} = \frac{\pi^2}{24}$

$S_{odd} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ... \infty$

$= S - S_{even}$

$= \frac{\pi^2}{6} - \frac{\pi^2}{24} = \frac{3\pi^2}{24} = \frac{\pi^2}{8}$

7. $\{a_n\}$ and $\{b_n\}$ are two sequences given by $(x)^{1/2^n} + (y)^{1/2^n}$ and $b_n = (x)^{1/2^n} - (y)^{1/2^n}$ for all n $\in$ N. The value of $a_1 a_2 a_3$......an is equal to

(A) $x - y$ (B) $\frac{x+y}{b_n}$ (C) $\frac{x-y}{b_n}$ (D) $\frac{xy}{b_n}$

Sol. C

$a_n = x^{1/2^n} + y^{1/2^n}$ & $b_n = x^{1/2^n} - y^{1/2^n}$, $\forall\, n \in N$

$\Rightarrow \quad a_n b_n = (x^{1/2^n})^2 - (y^{1/2^n})^2$

$\Rightarrow \quad a_n b_n = x^{1/2^{n-1}} - y^{1/2^{n-1}} = b_{n-1}$

$$a_1 a_2 a_3 \ldots a_n \times \frac{b_1 b_2 b_3 \ldots b_n}{b_1 . b_2 . b_3 \ldots b_n}$$

$$= \frac{(a_1b_1)(a_2b_2)(a_3b_3)\ldots(a_nb_n)}{b_1 . b_2 . b_3 \ldots b_n}$$

$$= \frac{(x-y)(b_1)(b_2)(b_3)\ldots(b_{n-1})}{b_1 b_2 b_3 \ldots b_{n-1} . b_n} = \frac{x-y}{b_n}$$

8. If the angles of a triangle are in the ratio 4 : 1 : 1, then the ratio of the longest side to the perimeter is

(A) $\sqrt{3} : (2+\sqrt{3})$ (B) $1 : \sqrt{3}$ (C) $1 : 2+\sqrt{3}$ (D) 2 : 3

9. The binomial expansion of $\left(x^k + \frac{1}{x^{2k}}\right)^{3n}$, $n \in N$ contains a term independent of x

(A) only if k is an integer
(B) only if k is a natural number
(C) only if k is rational
(D) for any real k

Sol. D

$\left(x^k + \frac{1}{x^{2k}}\right)^{3n}$, $n \in N$ Independent of x

$T_{r+1} = {}^{3n}C_r (x^k)^{3n-r} \left(\frac{1}{x^{2k}}\right)^r$

$= {}^{3n}C_r\, x^{3nk - rk - 2kr} = {}^{3n}C_r\, x^{3k(n-r)}$

For Constant term $\Rightarrow 3k(n-r) = 0 \Rightarrow n = r$

$\therefore T_{r+1} = {}^{3n}C_n$ true for any real k or $K \in R$

10. The general solution of the equation $\tan x + \tan\left(x + \frac{\pi}{3}\right) + \tan\left(x + \frac{2\pi}{3}\right) = 3$ is

(A) $\frac{n\pi}{4} + \frac{\pi}{12}, n \in I$
(B) $\frac{n\pi}{3} + \frac{\pi}{6}, n \in I$
(C) $\frac{n\pi}{3} + \frac{\pi}{12}, n \in I$
(D) None of these

Sol. c

$$\tan x + \tan\left(x+\frac{\pi}{3}\right) + \tan\left(x+\frac{2\pi}{3}\right) = 3$$

$$\Rightarrow \tan x + \frac{\tan x+\sqrt{3}}{1-\sqrt{3}\tan x} + \frac{\tan x-\sqrt{3}}{1+\sqrt{3}\tan x} = 3$$

$$\Rightarrow \frac{\tan x - 3\tan^3 x + \tan x + \sqrt{3}\tan^2 x + \sqrt{3} + 3\tan x + \tan x - \sqrt{3}\tan^2 x - \sqrt{3} + 3\tan x}{1-3\tan^2 x} = 3$$

$$\Rightarrow \frac{9\tan x - 3\tan^3 x}{1-3\tan^2 x} = 3 \Rightarrow \frac{3\tan x - \tan^3 x}{1-3\tan^2 x} = 1$$

$$\Rightarrow \tan 3x = 1 \Rightarrow 3x = n\pi + \frac{\pi}{4} \quad n \in I$$

$$\Rightarrow x = \frac{n\pi}{3} + \frac{\pi}{12} \quad n \in I$$

11. $\dfrac{\sin 3\theta}{2\cos 2\theta + 1} = \dfrac{1}{2}$ if

(A) $\theta = n\pi + \dfrac{\pi}{6}, n \in I$

(B) $\theta = 2n\pi - \dfrac{\pi}{6}, n \in I$

(C) $\theta = n\pi + (-1)^n \dfrac{\pi}{6}, n \in I$

(D) $\theta = n\pi - \dfrac{\pi}{6}, n \in I$

Sol. c

$$\frac{\sin 3\theta}{2\cos 2\theta + 1} = \frac{1}{2}$$

$$\Rightarrow \frac{3\sin\theta - 4\sin^3\theta}{2 - 4\sin^2\theta + 1} = \frac{1}{2}$$

$\Rightarrow$ $2\sin\theta\,[3 - 4\sin^2\theta] = (3 - 4\sin^2\theta)$

$\Rightarrow$ $(2\sin\theta - 1)(3 - 4\sin^2\theta) = 0$

$\Rightarrow$ $\sin\theta = \frac{1}{2} \Rightarrow \theta = n\pi + (-1)^n \frac{\pi}{6}, n \in I$

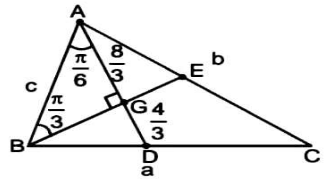
or $\sin^2\theta = \frac{3}{4} \Rightarrow \theta = n\pi \pm \frac{\pi}{3}, m \in I$

But doesn't satisfy the given equation

12. In a triangle ABC, medians AD and BE are drawn. If $AD = 4$, $\angle DAB = \dfrac{\pi}{6}$ and $\angle ABE = \dfrac{\pi}{3}$, then the area of the ΔABC is

(A) $\dfrac{8}{3}$ (B) $\dfrac{16}{3}$ (C) $\dfrac{32}{3\sqrt{3}}$ (D) $\dfrac{64}{3}$

Sol. **c**



AD = 4

In $\triangle ABG$, $\frac{\frac{8}{3}}{c} = \sin\frac{\pi}{3}$

$\Rightarrow C = \frac{8}{3}\ \frac{2}{\sqrt{3}} = \frac{16}{3\sqrt{3}}$

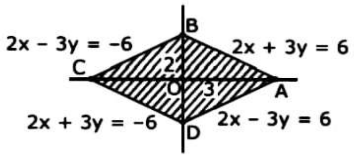
Area $\triangle ABD$ = Area $\triangle ADC$

$\triangle ABC = 2\triangle ABD = 2 \times \frac{1}{2}$ (AD) (AB) $\sin\frac{\pi}{6}$

$= 4 \cdot \frac{16}{3\sqrt{3}} \cdot \frac{1}{2} = \frac{32}{3\sqrt{3}}$ sq. units

13. The area enclosed by $2|x| + 3|y| \leq 6$ is
(A) 3 sq. units (B) 4 sq. units (C) 12 sq. units (D) 24 sq. units

Sol. **c**
$2|x| + 3|y| \leq 6$
area ABCD = 4 ($\triangle$OAB)



$= 4\left(\frac{1}{2}.2 \times 3\right) = 12$ sq. units

14. $\sqrt{-1-\sqrt{-1-\sqrt{-1...\infty}}}$ is equal to, where ω is the imaginary cube of root of unity and $i = \sqrt{-1}$.
(A) ω and ω^2 (B) $-\omega$ and $-\omega^2$ (C) $1+i$ or $1-i$ (D) $-1+i$ or $-1-i$

15. If a flagstaff subtends equal angles at the e points A, B, C and D on the horizontal ground through the foot of flagstaff, then the points A, B, C and D necessarily form a
(A) Rectangle (B) Parallelogram (C) Square **(D)** None of these

Sol.

16. If $i = \sqrt{-1}$, then $4 + 5\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{334} + 3\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{365}$ is equal to
(A) $1-i\sqrt{3}$ (B) $-1+i\sqrt{3}$ **(C)** $i\sqrt{3}$ (D) $-i\sqrt{3}$

17. If $x_1\, x_2\, x_3$ as well as y_1, y_2, y_3, are in G.P. with the same common ratio, then the points $(x_1, y_1\})$, (x_2, y_2) and (x_3, y_3)
(a) lie on a straight line (b) lie on an ellipse
(c) lie on a circle (d) are vertices of a triangle

18. If in the expansion of $(1 + x)^m (1 - x)^n$, the coefficients of x and x^2 are 3 and - 6 respectively, then m is
(a) 6 (b) 9 **(c)** 12 (d) 24

19. Consider an infinite geometric series with first term a and common ratio r. If its sum is 4 and the second term is $\frac{3}{4}$, then

(A) $a=\frac{4}{7}, r=\frac{3}{7}$ (B) $a=2, r=\frac{3}{8}$ (C) $a=\frac{3}{2}, r=\frac{1}{2}$ **(D)** $a=3, r=\frac{1}{4}$

20. If the line $x - 1 = 0$ is the directrix of the parabola $y^2 - kx + 8 = 0$ then one of the values of k is

(A) $\frac{1}{8}$ (B) 8 **(C)** 4 (D) $\frac{1}{4}$

21. The number of ways to fill each of of the four cells of the table with a distinct natural number, such that the sum of the numbers is 10 and the sum of the numbers places diagonally are equal is

Sol. The natural numbers are 1, 2, 3 and 4 (any others would lead to the sum exceeding 10)

Clearly, in one diagonal we have to place 1, 4 And in the other 2, 3

So we first select a diagonal, and then we arrange the numbers in its boxes

Number of ways of selection of the diagonal $= 2!$

The number of arrangements of numbers$= 2! \times 2! \times 2! = 8$.

22. Number of ways in which 5 boys and 4 girls can be arranged on a circular table such that no two girls sit together and two particular boys are always together.

Ans. 288

23. Consider the equation $x^2 + 2x - n = 0$, where $n \in N$ and $n \in [5, 100]$. The total number of different values of n so that the given equation has integral roots is

Sol. We have,

$x^2 + 2x - n = 0$

We know that for a Quadratic equation

$ax^2 + bx + c = 0,\ a \neq 0$ the roots are $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$.

$\Rightarrow x = -1 \pm \sqrt{n+1}$

Thus, $n + 1$ should be a perfect square for integral roots.

Now,

$n \in [5, 100] \Rightarrow n + 1 \in [6, 101]$

Perfect square values of $n + 1$ are 9, 16, 25, 36, 49, 64, 81, 100

Hence, number of values is 8.

24. If $\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha = \cot\alpha - n\cot n\alpha$ then value of n is

Sol. Consider $\cot\alpha - \tan\alpha = \frac{1-\tan^2\alpha}{\tan\alpha} = 2\cot 2\alpha \quad \ldots(1)$

Given relation can be written as

$n\cot n\alpha = \cot\alpha - \tan\alpha - 2\tan 2\alpha - 4\tan 4\alpha$

$= 2\cot(2\alpha) - 2\tan 2\alpha - 4\tan 4\alpha$

$= 2(2\cot 4\alpha) - 4\tan 4\alpha$

$= 4[\cot 4\alpha - \tan 4\alpha]$

$= 4(2\cot 8\alpha)$

$= 8\cot(8\alpha)$

$\therefore n = 8$

25. The expression $(x + (x^3 - 1)^{1/2})^5 + (x - (x^3 - 1)^{1/2})^5$ is a polynomia degree

ans - 7

26. Let x be the arithmetic mean and y, z be the two geometric means between any two positive numbers. Then $\frac{y^3+z^3}{xyz} =$ _______.

ans - 2

27. For integer n > 1, the digit at unit's place in the number $\sum_{r=0}^{100} r! + 2^{2^n}$ is

ans - 0

28. The letters of the word 'MEERUT' are arranged in all possible ways as in a dictionary, then the rank of the word 'MEERUT' is

ans - 122

29. Number of terms free from radical sign in the expansion of $(1 + 3^{1/3} + 7^{1/7})$ 10 is

Sol. **c**

$3^{1/3}$	$7^{1/7}$	1
0	0	10
3	0	7
6	0	4
9	0	1
3	7	0
0	7	3

$\therefore$ no. of terms are 6

30. The area of the quadrilateral formed by the tangents at the end points of latus recta to the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1,$ is

Ans - 27