## PHYSICS

## SECTION 1 (Maximum Marks: 12)

- This section contains THREE (03) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks $\quad:+4$ ONLY if (all) the correct option(s) is(are) chosen;
Partial Marks $\quad:+3$ If all the four options are correct but ONLY three options are chosen;
Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;
Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks :-2 In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
choosing ONLY (A), (B) and (D) will get +4 marks;
choosing ONLY (A) and (D) will get +2 marks;
choosing ONLY (A) will get +1 mark; choosing ONLY (D) will get +1 mark; choosing no option (i.e. the question is unanswered) will get 0 marks; and choosing any other combination of options will get -2 marks.

1. A slide with a frictionless curved surface, which becomes horizontal at its lower end, is fixed on the terrace of a building of height 3 h from the ground, as shown in the figure. A spherical ball of mass $m$ is released on the slide from rest at a height $h$ from the top of the terrace. The ball leaves the slide with a velocity $\vec{u}_{0}=u_{0} \hat{x}$ and falls on the ground at a distance $d$ from the building making an angle $\theta$ with the horizontal. It bounces off with a velocity $\vec{v}$ and reaches a maximum height $h_{1}$. The acceleration due to gravity is $g$ and the coefficient of restitution of the ground is $1 / \sqrt{3}$. Which of the following statement(s) is(are) correct?

(a) $d=2 \sqrt{3} h_{1}$
(b) $\overrightarrow{\mathrm{v}}=\sqrt{2 g h} \hat{x}+\sqrt{6 g h} \hat{z}$
(c) $\theta=30^{\circ}$
(d) $h_{1}=h$

Sol.


$$
\vec{u}_{0}=\sqrt{2 g h} \hat{x}
$$

Using energy conservation :-

$$
\begin{aligned}
& \frac{1}{2} m u_{0}^{2}+3 m g h=\frac{1}{2} m v_{1}^{2} \Rightarrow v_{1}=\sqrt{8 g h} \\
& \sqrt{u_{0}^{2}+v_{1}^{2}}=\sqrt{8 g h} \Rightarrow\left|\vec{v}_{1}\right|=\sqrt{6 g h} \\
& d=u_{0} \times T \text { and } \mathrm{T}=\sqrt{\frac{6 \mathrm{~h}}{\mathrm{~g}}} \Rightarrow d=2 \sqrt{3} h
\end{aligned}
$$

After collision velocity changes only along Z - direction


Using energy conservation
$\frac{1}{2} m \nu^{2}=\frac{1}{2} m u_{0}{ }^{2}+m g h_{1}$
$\Rightarrow h_{1}=h$
2. A small ball is connected to a block by a light string of length 1 . Both are initially on the ground. There is sufficient friction on the ground to prevent the block from slipping. The ball is projected vertically up with a velocity u , where $2 \mathrm{gl}<\mathrm{u}^{2}<3 \mathrm{gl}$ when string is making angle $\theta$ with the horizontal and $\left|v_{x}\right|$ magnitude of the horizontal component of the velocity at that point. The centre of mass of the 'block + ball' system is C.

(a) C will move along a circle.
(b) $\left|v_{x}\right|$ will be maximum when $\sin \theta=\frac{\mathrm{u}^{2}}{3 g l}$
(c) $\left|v_{x}\right|$ will be maximum when $\cos \theta=\frac{\mathrm{u}^{2}}{2 g l}$
(d) $\left|v_{x}\right|$ will first increase and then decrease

Sol. $\quad \therefore 2 \mathrm{gl}<\mathrm{u}^{2}<3 \mathrm{gl} \Rightarrow \theta<\frac{\pi}{2}$


As the block does not move, the ball moves along a circular path of radius $l$. The centre of mass of the system always lies somewhere on the string.
Let $v=$ speed of ball when the string makes an angle $\theta$ with the horizontal.

$$
\frac{1}{2} m v^{2}=\frac{1}{2} m u^{2}-m g l \sin \theta
$$

The horizontal component of $v=V=v \sin \theta=\sin \theta \sqrt{u^{2}-2 g l \sin \theta}$.
For $V$ to be maximum, $\frac{d V}{d \theta}=0$, which gives $\sin \theta=\frac{u^{2}}{3 g l}$.
3. The escape velocity for a planet is $v_{\mathrm{e}}$. A particle is projected from its surface with a speed $v$. For this particle to move as a satellite around the planet,
(a) $2 v_{e}<2 v<\sqrt{2} v_{e}$
(b) $\frac{v_{e}}{\sqrt{2}}<v<v_{e}$
(c) $v_{e}<v<\sqrt{2} v_{e}$
(d) $\sqrt{2} v_{e}<2 v<2 v_{e}$

Sol. For a satellite orbiting very close to the earth's surface, the orbital velocity

$$
=\sqrt{R g}\left(\because m g=\frac{m v^{2}}{R}\right) \text {.This is equal to the velocity of projection and is the minimum velocity }
$$

required to go into orbit. Also, the satellite would escape completely and not go into orbit for $v \geq v_{e}$
$\therefore \frac{v_{e}}{\sqrt{2}}<v<v_{\text {e }}$

## SECTION 2 (Maximum Marks: 12)

- This section contains FOUR (04) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks $\quad:+3$ If ONLY the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : -1 In all other cases.
4. A bar of mass $\mathrm{M}=1.00 \mathrm{~kg}$ and length $\mathrm{L}=0.20 \mathrm{~m}$ is lying on a horizontal frictionless surface. One end of the bar is pivoted at a point about which it is free to rotate. A small mass $\mathrm{m}=0.10 \mathrm{~kg}$ is moving on the same horizontal surface with $5.00 \mathrm{~m} \mathrm{~s}^{-1}$ speed on a path perpendicular to the bar. It hits the bar at a distance $\mathrm{L} / 2$ from the pivoted end and returns back on the same path with speed v . After this elastic collision, the bar rotates with an angular velocity $\omega$. Which of the following statement is correct?
(A) $\omega=6.98 \mathrm{rad} \mathrm{s}^{-1}$ and $\mathrm{v}=4.30 \mathrm{~m} \mathrm{~s}^{-1}$
(B) $\omega=3.75 \mathrm{rad} \mathrm{s}^{-1}$ and $\mathrm{v}=4.30 \mathrm{~m} \mathrm{~s}^{-1}$
(C) $\omega=3.75 \mathrm{rad} \mathrm{s}^{-1}$ and $\mathrm{v}=10.0 \mathrm{~m} \mathrm{~s}^{-1}$
(D) $\omega=6.80 \mathrm{rad} \mathrm{s}^{-1}$ and $\mathrm{v}=4.10 \mathrm{~m} \mathrm{~s}^{-1}$

Sol.

$$
\begin{aligned}
& \Rightarrow \quad \frac{L}{2} \omega+v=v_{0} \quad e_{1}-e^{n} \text {-(2) } \\
& \text { have } m=0.1 \mathrm{~kg}, M=1 . \mathrm{kg}, \quad L=0.2 \mathrm{~m} \\
& \text { solving } \mathrm{H}^{4} \text { (1) f(2) } \\
& \text { correction option is A. }
\end{aligned}
$$

5. Two satellites P and Q are moving in different circular orbits around the Earth (radius R). The heights of P and $Q$ from the Earth surface are $h_{P}$ and $h_{Q}$, respectively, where $h_{P}=R / 3$. The accelerations of $P$ and $Q$ due to Earth's gravity are $g_{p}$ and $g_{Q}$, respectively. If $g_{p} / g_{Q}=36 / 25$, what is the value of $h_{Q}$ ?
(A) $3 \mathrm{R} / 5$
(B) $\mathrm{R} / 6$
(C) $6 \mathrm{R} / 5$
(D) $5 \mathrm{R} / 6$

Sol.

$$
\begin{aligned}
& \text { Given } \mathrm{hp}_{\mathrm{p}}=R / 3 \\
& \text { gravitational } a C C l^{n} \text { at height } h^{\prime}=-\frac{G M}{(R+h)^{2}} \\
& g_{p}=\frac{9 G M}{16 R^{2}} \quad \& \quad g_{q}=\frac{G M}{(R+h q)^{2}} \\
& \frac{g_{p}}{g_{q}}=\frac{q_{1}+4}{16 R^{2}} \times \frac{(R+h a)^{2}}{s^{2} \pi_{1}}=\frac{z_{6}^{4}}{25} \\
& \Rightarrow \quad \frac{R+h_{q}}{R}=\frac{8}{5} \Rightarrow h_{q}=\frac{3 R}{5} \\
& \text { Correct option is (A) }
\end{aligned}
$$

6. A uniform rod of mass $m$, length $L$, area of cross-section A and Young's modulus $Y$ hangs from the ceiling. Its elongation under its own weight will be
(a) Zero
(b) $\frac{m g L}{2 A Y}$
(c) $\frac{m g L}{A Y}$
(d) $\frac{2 m g L}{A Y}$

Sol. Mass of section $B C=\frac{m}{L}(L-y)$.
$\therefore \quad$ tension at $\mathrm{B}=T=\frac{m}{L}(L-y) g$.
$\therefore \quad$ elongation of element $d y$ at B

$$
=d x=(d y) \frac{T}{A Y}=\frac{m}{L}(L-y) g \frac{d y}{A Y} .
$$



Total elongation $=\int d x=\frac{m g}{L A Y} \int_{0}^{L}(L-y) d y=\frac{m g L}{2 Y A}$.
7. A homogeneous solid cylinder of length $L$ and cross-sectional area $4 / 5$ is immersed such that it floats with its axis vertical at the liquid-liquid interface with length $\mathrm{L} / 4$ in the denser liquid as shown in the figure. The lower density liquid is open to atmosphere having pressure $\mathrm{p}_{0}$. Then, density D of solid is given by

(a) $\frac{5}{4} d$
(b) $\frac{4}{5} d$
(c) $4 d$
(d) $\frac{d}{5}$

Sol.


Weight of the cylinder $=$ force of buoyancy on the cylinder due to upper liquid + force of buoyancy on the cylinder due to lower liquid

$$
\begin{gathered}
\therefore \quad(A / 5)(L) D g=(A / 5)(3 L / 4)(d) g \\
\therefore \quad+(A / 5)(L / 4)(2 d)(g) \\
\therefore=\left(\frac{3}{4}\right) d+\left(\frac{1}{4}\right)(2 d) \\
D=\frac{5}{4} d
\end{gathered}
$$

## SECTION 3 (Maximum Marks: 24)

- This section contains SIX (06) questions.
- The answer to each question is a NON-NEGATIVE INTEGER.
- For each question, enter the correct integer corresponding to the answer on OMR sheet.

Answer to each question will be evaluated according to the following marking scheme:
Full Marks $\quad:+4$ If ONLY the correct integer is entered;
Zero Marks : 0 In all other cases.
8. Distance between the centres of two stars is 10 a . The masses of these stars are M and 16 M and their radii $a$ and $2 a$ respectively. A body of mass $m$ is fired straight from the surface of the larger star towards the surface of the smaller star. If $\mathrm{V}_{\min }$ is the minimum initial speed to reach the surface of the smaller star. If

$$
\mathrm{N}=\left(2 \times \sqrt{\frac{5 \mathrm{a}}{\mathrm{GM}}}\right) \mathrm{V}_{\min } . \text { Calculate } \mathrm{N}=
$$

$\qquad$ .

Sol.


Let ' $p$ ' be the point where total gravitational field become Zen:-

$$
\begin{aligned}
& \frac{4 M}{r_{2}^{2}}=\frac{16 G M}{\left(6 r_{2}\right)^{2}} \Rightarrow \frac{r_{1}}{}=4 r_{2} \\
& \& r_{1}+r_{2}=10 a \Rightarrow r_{2}=2 a \\
& r_{1}=8 a_{1}
\end{aligned}
$$

Now if mass $m$ can reach upto ' p ' then it will get attracted towards smaller star. Using Mechanical energy conservation :-

$$
\begin{aligned}
& \frac{1}{2} m v_{m i n}^{2}-\frac{G m 16 M}{2 a}-\frac{G m m}{G a}=-\frac{16 G M m}{8 a}-\frac{4 M m}{2 a} \\
& \frac{1}{2} m v_{m i n}^{2} \\
& v_{\min }=\frac{G m M}{a}\left[8+\frac{1}{8}-2-\frac{1}{2}\right] \\
& N=\sqrt{\frac{45}{4} \frac{G M}{a}}=\frac{3}{2} \sqrt{\frac{5 G M}{a}} \\
& W \sqrt{\frac{54}{G M}} 4 \sqrt{\frac{5 G M}{a}} \times \frac{3}{x} \\
& H=15
\end{aligned}
$$

9. A cart is moving along $x$-direction with a velocity of $8 \mathrm{~m} / \mathrm{s}$. A person on the cart throws a stone with a velocity of $12 \mathrm{~m} / \mathrm{s}$ relative to himself. In the frame of reference of the cart the stone is thrown in $y$-z plane making an angle of $30^{\circ}$ with vertical z -axis. At the highest point of its trajectory the stone hits an object of equal mass hung vertically from branch of a tree by means of a string of length L. A completely inelastic collision occurs, in which the stone gets embedded in the object. If length $L$ of the string such that tension in the string becomes zero when the string becomes horizontal during the subsequent motion of the combined mass. Calculate $4 \times \mathrm{L}=$ $\qquad$ $\mathrm{m} .\left(\mathrm{g}=10 \mathrm{~m} / \mathrm{s}^{2}\right)$
Sol. $\quad \overrightarrow{\mathrm{V}}_{\mathrm{m}_{\mathrm{g}}}=8 \hat{x}+12 \cos 60^{\circ} \hat{y}+12 \sin 60^{\circ} \hat{z}$
At the heightest point of the trajectory velocity component along z-axis will become zero

$$
\left|\overrightarrow{\mathrm{V}}_{\mathrm{m}^{\prime} \mathrm{g}}\right|=10 \mathrm{~m} / \mathrm{sec}
$$

Let $\mathrm{V}_{1}$ be the velocity of combined mass after the collision
Using momentum conservation

$$
10 m=2 m \mathrm{~V}_{1} \Rightarrow \mathrm{~V}_{1}=5 \mathrm{~m} / \mathrm{sec}
$$

Using energy conservation

$$
\frac{1}{2} 2 m \mathrm{~V}_{1}^{2}=2 m g l \Rightarrow l=\frac{\mathrm{V}_{1}^{2}}{2 g} \Rightarrow 4 l=5 \mathrm{~m}
$$

10. Three objects $\mathrm{A}, \mathrm{B}$ and C are kept in a straight line on a frictionless horizontal surface. These have masses $\mathrm{m}, 2 \mathrm{~m}$ and m , respectively. The object A moves towards B with a speed $9 \mathrm{~ms}^{-1}$ and makes an elastic collision with it. Thereafter, B makes completely inelastic collision with C. All motions occur on the same straight line. Find the final speed of the object $\mathrm{C}=$ $\qquad$ (in ms ${ }^{-1}$ )

| $m$ | $2 m$ | $m$ |
| :---: | :---: | :---: |
| $A$ | $B$ | $C$ |

Sol. Let $\mathrm{V}_{\mathrm{B}} \& \mathrm{~V}_{\mathrm{A}}$ be the velocity of blocks after $1^{\text {st }}$ collision :-

$$
\begin{aligned}
& 9 \mathrm{~m}=2 \mathrm{mV}_{\mathrm{B}}+\mathrm{mV}_{\mathrm{A}} \& \frac{1}{2} m \times 81=\frac{1}{2} \times 2 m \mathrm{~V}_{\mathrm{B}}^{2}+\frac{1}{2} m \mathrm{~V}_{\mathrm{A}}^{2} \\
& \mathrm{~V}_{\mathrm{A}}=\left(9-2 \mathrm{~V}_{\mathrm{B}}\right) \ldots . . \mathrm{eq}^{\mathrm{n}}(\mathrm{i}) \\
& 2 \mathrm{~V}_{\mathrm{B}}^{2}+\mathrm{V}_{\mathrm{A}}^{2}=81 \ldots . \mathrm{eq}^{\mathrm{n}} \text { (ii) } \\
& \text { fromeq }{ }^{\mathrm{n}} \text { (i) \& (ii) } \mathrm{V}_{\mathrm{B}}=6 \mathrm{~m} / \mathrm{sec} \\
& \text { Using momentum conservation for blocks } \mathrm{B} \& \mathrm{C} \\
& 12 \mathrm{~m}=3 \mathrm{mV}{ }_{\mathrm{c}} \Rightarrow \mathrm{~V}_{\mathrm{c}}=4 \mathrm{~m} / \mathrm{sec}
\end{aligned}
$$

11. A uniform circular disc of mass 50 kg and radius 0.4 m is rotating with an angular velocity of $10 \mathrm{rad} / \mathrm{s}$ about its own axis, which is vertical. Two uniform circular rings, each of mass 6.25 kg and radius 0.2 m , are gently placed symmetrically on the disc in such a manner that they are touching each other along the axis of the disc and are horizontal. Assume that the friction is large enough such that the rings are at rest relative to the disc and the system rotates about the original axis. The new angular velocity (in rad s${ }^{-1}$ ) of the system is $\qquad$ .

Sol.

$$
\begin{aligned}
& \text { Using Conservation of Angular Momentum:- } \\
& I_{1} \infty_{1}=\left(I_{1}+2 I_{2}\right) \omega_{2} \\
& \text { here } I_{1}=\frac{M R^{2}}{2} \quad \& I_{2}=2 M r^{2} \\
& \omega_{2}=\frac{\left(M R^{2} / 2\right) \omega_{1}}{\left(\frac{M R^{2}}{2}+4 M r_{0}^{2}\right)} \Rightarrow \omega_{2}=8 \mathrm{rad} / \mathrm{sec}
\end{aligned}
$$

12. A particle of mass $m=1 \mathrm{~kg}$ moving in a circular path of radius $R=3 \mathrm{~m}$ with a constant speed $\mathrm{v}_{2}=\frac{11}{7} \mathrm{~m} / \mathrm{sec}$ is located at point $(2 R, 0)$ at time $t=0$ and a man starts moving with a velocity $v_{1}=2 \mathrm{~m} / \mathrm{sec}$ along the positive z -axis from origin at time $\mathrm{t}=0$. If $|\overrightarrow{\mathrm{L}}|$ be the linear momentum of the particle w.r.t. man as a function of time at $t=2$. Calculate $|\overrightarrow{\mathrm{L}}|^{2} \times 196=$ $\qquad$ $\mathrm{kg}^{2} \mathrm{~m}^{2} / \mathrm{sec}^{2}$


Sol.


$$
\begin{aligned}
& \mathrm{V}_{2}=\omega R \Rightarrow \omega=\frac{\mathrm{V}_{2}}{\mathrm{R}} \Rightarrow \theta=\frac{\mathrm{V}_{2} t}{\mathrm{R}} \Rightarrow \theta=\frac{11 \times 2}{7 \times 3}=\frac{22}{7} \times \frac{1}{3}=\frac{\pi}{3} \\
& \left(\overrightarrow{\mathrm{~V}}_{2}\right)_{\text {groomed }}=\left(-\mathrm{V}_{2} \sin \frac{\pi}{3} \hat{i}+\mathrm{V}_{2} \cos \frac{\pi}{3} \hat{k}\right) \Rightarrow-\frac{11 \sqrt{3}}{14} \hat{i}+\frac{11}{14} \hat{k} \&\left(\overrightarrow{\mathrm{~V}}_{1}\right)_{\text {groomed }}=2 \hat{k} \\
& \overrightarrow{\mathrm{~V}}_{\mathrm{p} / \mathrm{m}}=\left(\overrightarrow{\mathrm{V}}_{2}\right)_{\text {ground }}-\left(\overrightarrow{\mathrm{V}}_{1}\right)_{\text {ground }} \\
& \overrightarrow{\mathrm{V}}_{\mathrm{p} / \mathrm{m}}=\left(\overrightarrow{\mathrm{V}}_{2}\right)_{\text {ground }}-\left(\overrightarrow{\mathrm{V}}_{1}\right)_{\text {ground }} \Rightarrow-\frac{11 \sqrt{3}}{14} \hat{i}+\left(\frac{11}{14}-2\right) \hat{k}=-\frac{11 \sqrt{3}}{14} \hat{i}-\frac{17}{14} \hat{k} \\
& \vec{L}_{p / m}=m \times \overrightarrow{\mathrm{V}}_{\mathrm{p} / \mathrm{m}} \Rightarrow \mid \overrightarrow{\mathrm{L}}^{2} \times 196=17^{2}+(11 \sqrt{3})^{2}=652 \mathrm{~kg}^{2} \mathrm{~m}^{2} / \mathrm{sec}^{2}
\end{aligned}
$$

13. In the figure masses $m_{1}, m_{2}$ and M are $20 \mathrm{~kg}, 5 \mathrm{~kg}$ and 50 kg respectively. The coefficient of friction between $M$ and ground is zero. The coefficient of friction between $m_{1}$ and $M$ and that between $m_{2}$ and ground is 0.3 . The pulleys and the strings are massless. The string is perfectly horizontal between $\mathrm{P}_{1}$ and $\mathrm{m}_{1}$ and also between $\mathrm{P}_{2}$ and $\mathrm{m}_{2}$.
The string is perfectly vertical between $\mathrm{P}_{1}$ and $\mathrm{P}_{2}$. An external horizontal force F is applied to the mass M . Let the magnitude of the force of friction between $m_{1}$ and $M$ be $f_{1}$ and that between $m_{2}$ and the ground be $f_{2}$. For a perticular force $F$ it is found that $f_{1}=2 f_{2}$. Calculate $f_{1}+f_{2}+F=$ $\qquad$ N. (Take $\mathrm{g}=10 \mathrm{~m} / \mathrm{s}^{2}$.)


Sol.

$$
\begin{aligned}
& f_{1}(\text { max })=60 \mathrm{~N} \quad f_{2 \text { (mar) }}=15 \mathrm{~N} \\
& \text { since string is perfectly horizontal between } P_{1} \text { ff, } \\
& \text { \& } P_{2} \& \mathrm{H}_{2} \\
& \Rightarrow f_{2} \text { is at it's Limiting value. } \\
& \text { hence using } f_{1}=2 f_{2} \Rightarrow f_{1}=30 \mathrm{~N}<f_{1, \mathrm{maro}} \\
& \text { FD }^{\prime} \\
& T . \leftarrow m_{1} \longrightarrow \nrightarrow \text {, } \\
& \underset{i_{2}}{ } \rightarrow T \\
& \text { } 6 \\
& \text { All masses will move with same acceleition } \\
& \text { in the direction of applied force. } \\
& f_{1}-T=m_{1} a \mid T-f_{2}=m_{2} a \text { \& } F-f_{1}=M a \cdot \\
& 30 N-T=\begin{array}{c}
20 a \\
-(1)
\end{array} \left\lvert\, T-15 \mathrm{~N}=\begin{array}{c}
5 a \\
\text {-(2) }
\end{array} \quad R-30=\begin{array}{c}
50 a \\
-(3)
\end{array}\right. \\
& F=60 \mathrm{NM} \\
& f_{1}+f_{2}+F \text { 21 }(30+15+60) \\
& =105 \mathrm{NH}
\end{aligned}
$$

## SECTION 4 (Maximum Marks: 12)

- This section contains FOUR (04) Matching List Sets.
- Each set has ONE Multiple Choice Question.
- Each set has TWO lists: List-I and List-II.
- List-I has Four entries (P), (Q), (R) and (S) and List-II has Five entries (1), (2), (3), (4) and (5).
- FOUR options are given in each Multiple Choice Question based on List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks $\quad:+3$ ONLY if the option corresponding to the correct combination is chosen;
Zero Marks $\quad: \quad 0$ If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : - 1 In all other cases.
14. Column II gives five arrangements of two blocks $A$ and $B$. In each arrangement their masses are 3 kg and 2 kg respectively. Match the entries in column I with arrangements in column II.

## Column I

(A) Force on $B$ w.r.t. $A$ is equal to force on $A$ w.r.t. $B$

## Column II

(p)


(r)

(s)

(t)

(a) $\mathrm{A} \rightarrow \mathrm{t}, \mathrm{B} \rightarrow(\mathrm{p}, \mathrm{s}), \mathrm{C} \rightarrow(\mathrm{q}, \mathrm{s}), \mathrm{D} \rightarrow(\mathrm{p}, \mathrm{q}, \mathrm{s})$
(b) $\mathrm{A} \rightarrow(\mathrm{r}, \mathrm{t}), \mathrm{B} \rightarrow(\mathrm{p}, \mathrm{s}), \mathrm{C} \rightarrow(\mathrm{p}, \mathrm{q}, \mathrm{s}), \mathrm{D} \rightarrow(\mathrm{p}, \mathrm{q})$
(c) $\mathrm{A} \rightarrow \mathrm{r}, \mathrm{B} \rightarrow(\mathrm{q}, \mathrm{s}), \mathrm{C} \rightarrow(\mathrm{p}, \mathrm{q}), \mathrm{D} \rightarrow(\mathrm{p}, \mathrm{s})$
(d) $\mathrm{A} \rightarrow(\mathrm{r}, \mathrm{t}), \mathrm{B} \rightarrow(\mathrm{p}, \mathrm{q}), \mathrm{C} \rightarrow(\mathrm{p}, \mathrm{s}), \mathrm{D} \rightarrow(\mathrm{q}, \mathrm{s})$

Sol. Sol. Answer $A(r, t), B(p, s), C(p, q, s), D(p, q)$
(p) $a=\left(\frac{m_{A}-m_{B}}{m_{A}+m_{B}}\right) g=2 \mathrm{~m} / \mathrm{s}^{2}$
$\left|\bar{F}_{A B}\right|=12 \mathrm{~N}$ and $\left|\bar{F}_{B A}\right|=8 \mathrm{~N}$
$F_{B}=4 \mathrm{~N}$
(a) $a_{A}=\frac{18-f r}{3}=2 \mathrm{~m} / \mathrm{s}^{2}$
$a_{B}=\frac{16-f r}{2}=6 \mathrm{~m} / \mathrm{s}^{2}$
$F_{B A}=(6-2) \times 2=8 \mathrm{~N}$
(r) $a_{A}=a_{B}=0$
$\left|\vec{F}_{A B}\right|=\left|\vec{F}_{B C}\right|=0$
(s) $a=\frac{30-f r}{5}=2 \mathrm{~m} / \mathrm{s}^{2}$
(t) $a_{A}=a_{B}=0$
15. The potential energy for a conservative system is given by $U=\frac{x^{3}}{3}-\frac{5 x^{2}}{2}+6 x+3$. Match all the entries in Column I to all the entries in Column II.

> Column I
(A) The magnitude of net force on particle at $x=0$
(B) The equilibrium positions are
(C) The stable equilibrium position is
(D) Potential energy at unstable equilibrium
(a) $\mathrm{A} \rightarrow \mathrm{r}, \mathrm{B} \rightarrow(\mathrm{p}, \mathrm{r}), \mathrm{C} \rightarrow(\mathrm{p}, \mathrm{q}, \mathrm{r}), \mathrm{D} \rightarrow(\mathrm{p}, \mathrm{s})$
(b) $\mathrm{A} \rightarrow(\mathrm{p}, \mathrm{s}), \mathrm{B} \rightarrow(\mathrm{r}, \mathrm{q}), \mathrm{C} \rightarrow(\mathrm{q}), \mathrm{D} \rightarrow(\mathrm{p})$
(c) $\mathrm{A} \rightarrow \mathrm{s}, \mathrm{B} \rightarrow(\mathrm{q}, \mathrm{r}), \mathrm{C} \rightarrow(\mathrm{r}), \mathrm{D} \rightarrow(\mathrm{p})$
(d) $\mathrm{A} \rightarrow \mathrm{s}, \mathrm{B} \rightarrow(\mathrm{p}, \mathrm{r}), \mathrm{C} \rightarrow(\mathrm{s}), \mathrm{D} \rightarrow(\mathrm{q})$

## Column II

(p) $\frac{23}{3}$
(q) 2
(r) 3
(s) 6

Answer $A(s) ; B(q, r) ; C(r) ; D(p)$ $U=\frac{x^{3}}{3}-\frac{5 x^{2}}{2}+6 x+3$

Sol. $F=-\frac{d U}{d x}=-x^{2}+5 x-6$. At $x=0, F=-6$.
$\Rightarrow$ Magnitude is 6
$F=0 \Rightarrow x=3,2$. These are equilibrium position
16. Match the entries given in column I with those given in column II.

## Column I

(A) Plot of gravitational field due to a uniform solid sphere with distance from its centre.
(B) Plot of gravitational field due to a thin spherica shell with distance from centre.
(C) Plot of gravitational potential due to a uniform solid sphere with distance from centre.
(D) Plot of gravitational potential due to a thin spherical shell with distance from centre.
(a) $\mathrm{A} \rightarrow \mathrm{r}, \mathrm{B} \rightarrow(\mathrm{p}, \mathrm{r}), \mathrm{C} \rightarrow(\mathrm{p}, \mathrm{q}, \mathrm{t}), \mathrm{D} \rightarrow(\mathrm{p}, \mathrm{s})$
(b) $\mathrm{A} \rightarrow(\mathrm{p}, \mathrm{s}), \mathrm{B} \rightarrow(\mathrm{r}, \mathrm{q}), \mathrm{C} \rightarrow(\mathrm{q}), \mathrm{D} \rightarrow(\mathrm{p})$
(c) $\mathrm{A} \rightarrow \mathrm{s}, \mathrm{B} \rightarrow(\mathrm{q}, \mathrm{r}), \mathrm{C} \rightarrow(\mathrm{t}), \mathrm{D} \rightarrow(\mathrm{p})$
(d) $\mathrm{A} \rightarrow(\mathrm{p}, \mathrm{r}) \mathrm{B} \rightarrow(\mathrm{q}), \mathrm{C} \rightarrow(\mathrm{p}, \mathrm{s}, \mathrm{t}), \mathrm{D} \rightarrow(\mathrm{p}, \mathrm{r}, \mathrm{s})$

## Column II

(p) Continuos
(q) Discontinuos
(r) Straight line
(s) Rectangular hyperbola
(t) Parabola

Sol.
Answer $A(p, r), B(q), C(p, s, t), D(p, r, s)$
(B)

(C)

(D)

17. A uniform solid sphere is resting on a smooth horizontal floor. A particle hits the sphere at an angle $\theta$. Then for just before and after the collision match the coloumn


Column I
(A) $\mathrm{e}=1$
(B) $\mathrm{e}=0$
(C) Particle sticks
(D) $-\frac{\pi}{2}<\theta<0 \& e=1$

## Column II

(p) Linear momentum of the particle sphere system is conserved in horizontal direction
(q) Linear momentum of the particle- sphere system is conserved in the verticle direction
(r) Linear momentum of the particle in the direction tangential to the sphere at the point of collision is conserved with the sphere
(s) Angular momentum of particle sphere system about point $P$ is conserved
(t) Angular momentum of particle- sphere system about point O is conserved
(a) $\mathrm{A} \rightarrow(\mathrm{p}, \mathrm{r}), \mathrm{B} \rightarrow(\mathrm{p}, \mathrm{r}, \mathrm{t}), \mathrm{C} \rightarrow(\mathrm{p}, \mathrm{t}), \mathrm{D} \rightarrow(\mathrm{p}, \mathrm{q}, \mathrm{r}, \mathrm{s}, \mathrm{t})$
(b) $\mathrm{A} \rightarrow(\mathrm{p}, \mathrm{r}, \mathrm{t}), \mathrm{B} \rightarrow(\mathrm{p}, \mathrm{r}, \mathrm{t}), \mathrm{C} \rightarrow(\mathrm{p}, \mathrm{q}, \mathrm{t}), \mathrm{D} \rightarrow(\mathrm{p}, \mathrm{q}, \mathrm{r}, \mathrm{s}, \mathrm{t})$
(c) $\mathrm{A} \rightarrow \mathrm{s}, \mathrm{B} \rightarrow(\mathrm{q}, \mathrm{r}), \mathrm{C} \rightarrow(\mathrm{t}), \mathrm{D} \rightarrow(\mathrm{p})$
(d) $\mathrm{A} \rightarrow(\mathrm{p}, \mathrm{r}) \mathrm{B} \rightarrow(\mathrm{q}), \mathrm{C} \rightarrow(\mathrm{p}, \mathrm{s}, \mathrm{t}), \mathrm{D} \rightarrow(\mathrm{p}, \mathrm{r}, \mathrm{s})$

Sol.
Answer $A(p, r, t), B(p, r, t), C(p, q, t), D(p, q, r, s, t)$
If $\Sigma F=0$
$\Rightarrow$ Linear momentum is conserved.
$\Sigma \vec{\tau}=0$
Angular momentum is conserved.
When particle hits horizontally, there is a vertically imuplse from ground. So, momentum along vertical is not conserved. The torque of this impulse is zero about $O$, but not about $P$.

## CHEMISTRY

SECTION 1 (Maximum Marks: 12)

- This section contains THREE (03) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks $\quad:+4$ ONLY if (all) the correct option(s) is(are) chosen;
Partial Marks $\quad:+3$ If all the four options are correct but ONLY three options are chosen;
Partial Marks $\quad:+2$ If three or more options are correct but ONLY two options are chosen, both of which are correct;
Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : -2 In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then choosing ONLY (A), (B) and (D) will get +4 marks; choosing ONLY (A) and (B) will get +2 marks; choosing ONLY (A) and (D) will get +2 marks; choosing ONLY (B) and (D) will get +2 marks; choosing ONLY (A) will get +1 mark; choosing ONLY (B) will get +1 mark;
choosing ONLY (D) will get +1 mark;
choosing no option (i.e. the question is unanswered) will get 0 marks; and choosing any other combination of options will get -2 marks.

1. The halogen form compounds among themselves with formula $\mathrm{XX}^{\prime}, \mathrm{XX}_{3}^{\prime}, \mathrm{XX}_{5}^{\prime}$ and $\mathrm{XX}^{\prime}$, where X is the heavier halogen $\left[\mathrm{x} \neq \mathrm{x}^{\prime}\right]$. which of the following pairs representing their structures and being polar and nonpolar are correct?
(a) XX '-Linear - Polar.
(b) $\mathrm{XX}^{\prime}$ - -T -shaped-Polar.
(c) $\mathrm{XX}_{5}{ }_{5}$-square pyramidal-Non-polar.
(d) $\mathrm{XX}^{\prime}$ - - Pentagonal bipyramidal-Non-polar.
2. Buffer solution can be prepared by
(a) $\mathrm{CH}_{3} \mathrm{COO}^{-} \mathrm{Na}^{+}+\mathrm{CH}_{3} \mathrm{COOH}$
(b) $\mathrm{CH}_{3} \mathrm{COO}^{-} \mathrm{Na}^{+}+\mathrm{HCl}$ (L.R.)
(c) $\mathrm{NH}_{3}$ aq. $+\mathrm{NH}_{4} \mathrm{Cl}$
(d) $\mathrm{NH}_{3}+\mathrm{NaOH}$
3. For which of the following molecules is/are having non-zero dipole moment?
(a)

(b)

(c)

(d)


## SECTION 2 (Maximum Marks: 12)

- This section contains FOUR (04) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks $\quad:+3$ If ONLY the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : - 1 In all other cases.
4. If $2 \pi r_{n} \neq n \lambda$ for a microscopic particle, then what will happen-
(a) Emission of spectral lines will not take place.
(b) Concept of stationary orbit will be valid.
(c) Orbits will not exist.
(d) Energy will be absorbed from Surrounding
5. 300 ml of 0.2 M Pyridine solution is added with 200 ml of 0.2 M NaOH solution. Find degree of ionisation \& pH of the solution? $\left[\mathrm{K}_{\mathrm{b}}=5 \times 10^{-6}\right]$
(a) $6.25 \times 10^{-5}, \mathrm{pH}=12.9$
(b) $7.5 \times 10^{-6}, \mathrm{pH}=12$
(c) $6 \times 10^{-4}, \mathrm{pH}=10$
(d) $7.5 \times 10^{-2}, \mathrm{pH}=2$
6. Which one of the following Structures has the IUPAC name cyclohexylidene methanone?
(a)

(b)

(c)

(d)

7. 2 moles of $\mathrm{FeC}_{2} \mathrm{O}_{4}$ and 1 mole of $\mathrm{Fe}\left(\mathrm{C}_{2} \mathrm{O}_{4}\right)_{3}$ are present in a solution mixture. How many moles of $\mathrm{K}_{2} \mathrm{Cr}_{2} \mathrm{O}_{7}$ should be added to required to complete neutralization
(a) 4 mole
(b) 2 mole
(c) 1 mole
(d) 3 mole

## SECTION 3 (Maximum Marks: 24)

- This section contains SIX (06) questions.
- The answer to each question is a NON-NEGATIVE INTEGER.
- For each question, enter the correct integer corresponding to the answer on OMR sheet.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks $\quad:+4$ If ONLY the correct integer is entered;
Zero Marks : 0 In all other cases.
8. For the reaction $\mathrm{Ag}(\mathrm{CN})_{2}{ }^{-} \rightleftharpoons \mathrm{Ag}^{+}(\mathrm{aq})+2^{-} \mathrm{CN}(\mathrm{aq})$, the $\mathrm{K}_{\mathrm{c}}$ at $25^{\circ} \mathrm{c}$ is $4 \times 10^{-19} \mathrm{M}$. If the $\mathrm{Ag}^{+}(\mathrm{aq})$

Concentration in solution is $\mathrm{b} \times 10^{-18} \mathrm{M}$ which was originally 0.1 M KCN and $0.03 \mathrm{M} \mathrm{AgNO}_{3}$. The value of " 2 b " is $\qquad$ .
ans :-15
9. Total number of degree of unsaturation of the following compounds

ans :-11
10. On decreasing the pH from 7 to 2 , the solubility of a sparingly Soluble salt (MX) of a weak acid $(\mathrm{HX})$ increased from $10^{-4} \mathrm{M}$ to $10^{-3} \mathrm{M}$. The $\mathrm{pK}_{\mathrm{a}}$ of HX is $\qquad$ .
ans:-4
11. Calculate n -factor of sulpher ( S ) in the following reaction

$$
\mathrm{As}_{2} \mathrm{~S}_{3} \rightarrow \mathrm{H}_{3} \mathrm{AsO}_{4}+\mathrm{SO}_{4}^{-2}
$$

ans :-28
12. A single electron ion $\mathrm{M}^{x+}$ has electron in $\mathrm{N}^{\mathrm{th}}$ orbit. If energy of electron in $(x+1)^{\text {th }}$ orbit of hydrogen atom is $\left(\frac{3.4}{\mathrm{~N}^{2}}\right) \mathrm{eV}$ while velocity of electron in this $(x+1)$ th orbit of H -atom is $5.45 \times 10^{5} \mathrm{~m} / \mathrm{s}$. The value of $(x+\mathrm{N})$ is $\qquad$ .
Ans;- 5
13. Among $\left[\mathrm{I}_{3}\right]^{-},\left[\mathrm{I}_{3}\right]^{+}, \mathrm{XeO}_{2} \mathrm{~F}_{2}, \mathrm{SOCl}_{2}, \mathrm{XeF}_{4}, \mathrm{SF}_{4}, \mathrm{Ni}(\mathrm{CO})_{4}, \mathrm{XeOF}_{4}, \mathrm{SOCl}_{2}, \mathrm{IF}_{7}, \mathrm{XeF}_{2}, \mathrm{H}_{3} \mathrm{PO}_{3}, \mathrm{H}_{2} \mathrm{SO}_{4}$, the total number of species having $\mathrm{sp}^{3}$ hybridised central atom is $\qquad$ .
Ans;- 6

## SECTION 4 (Maximum Marks: 12)

- This section contains FOUR (04) Matching List Sets.
- Each set has ONE Multiple Choice Question.
- Each set has TWO lists: List-I and List-II.
- List-I has Four entries (P), (Q), (R) and (S) and List-II has Five entries (1), (2), (3), (4) and (5).
- FOUR options are given in each Multiple Choice Question based on List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks $\quad:+3$ ONLY if the option corresponding to the correct combination is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks :-1 In all other cases.
14. Collumn - I
(A) Salt of weak acid \& weak base
(B) Salt of weak acid \& strong base
(C) Salt of strong acid and strong base
(D) Salt of strong acid and weak base
(a) $\mathrm{A} \rightarrow \mathrm{q}, \mathrm{B} \rightarrow \mathrm{p}, \mathrm{C} \rightarrow \mathrm{s}, \mathrm{D} \rightarrow \mathrm{r}$
(b) $\mathrm{A} \rightarrow \mathrm{r}, \mathrm{B} \rightarrow \mathrm{p}, \mathrm{C} \rightarrow \mathrm{s}, \mathrm{D} \rightarrow \mathrm{q}$
(c) $\mathrm{A} \rightarrow \mathrm{p}, \mathrm{B} \rightarrow \mathrm{q}, \mathrm{C} \rightarrow \mathrm{s}, \mathrm{D} \rightarrow \mathrm{r}$
(d) $\mathrm{A} \rightarrow \mathrm{q}, \mathrm{B} \rightarrow \mathrm{p}, \mathrm{C} \rightarrow \mathrm{r}, \mathrm{D} \rightarrow \mathrm{s}$
15. Column - I

Column - II
(A) $\mathrm{As}_{2} \mathrm{~S}_{3} \rightarrow \mathrm{AsO}_{4}^{3-}+\mathrm{SO}_{4}^{2-}$
(p) 28
(B) $\mathrm{I}_{2} \rightarrow \mathrm{I}^{-}+\mathrm{IO}_{3}^{-}$
(q) 6
(C) $\mathrm{P}_{4} \rightarrow \mathrm{PH}_{3}+\mathrm{H}_{3} \mathrm{PO}_{3}$
(r) 1
(D) $\mathrm{H}_{3} \mathrm{PO}_{2}+\mathrm{NaOH} \rightarrow \mathrm{NaH}_{2} \mathrm{PO}_{2}+\mathrm{H}_{2} \mathrm{O}$
(s) $5 / 3$
(a) $\mathrm{A} \rightarrow \mathrm{s}, \mathrm{B} \rightarrow \mathrm{p}, \mathrm{C} \rightarrow \mathrm{q}, \mathrm{D} \rightarrow \mathrm{r}$
(b) $\mathrm{A} \rightarrow \mathrm{p}, \mathrm{B} \rightarrow \mathrm{s}, \mathrm{C} \rightarrow \mathrm{q}, \mathrm{D} \rightarrow \mathrm{r}$
(c) $\mathrm{A} \rightarrow \mathrm{q}, \mathrm{B} \rightarrow \mathrm{s}, \mathrm{C} \rightarrow \mathrm{p}, \mathrm{D} \rightarrow \mathrm{r}$
(d) $\mathrm{A} \rightarrow \mathrm{p}, \mathrm{B} \rightarrow \mathrm{q}, \mathrm{C} \rightarrow \mathrm{r}, \mathrm{D} \rightarrow \mathrm{s}$
16. Match in the increasing orders given in the column - I with the property(ies) given in the column-II

Column -I
(A) $\mathrm{Na}^{+}<\mathrm{F}^{-}<\mathrm{O}^{2-}<\mathrm{N}^{2-}$
(B) $\mathrm{Li}^{+}<\mathrm{Na}^{+}<\mathrm{K}^{+}<\mathrm{Rb}^{+}<\mathrm{Cs}^{+}$
(C) O $<$ S $<$ F $<$ Cl
(D) $\mathrm{Cl}^{-}<\mathrm{K}^{+}<\mathrm{Ca}^{2+}<\mathrm{Sc}^{3+}$

Column-II
(p) Electronegativity
(q) Nuclear charge
(r) Size
(s) Electron Affinity
(t) Ionisation energy
(a) $\mathrm{A} \rightarrow \mathrm{r}, \mathrm{B} \rightarrow \mathrm{p}, \mathrm{s}, \mathrm{C} \rightarrow \mathrm{p}, \mathrm{D} \rightarrow \mathrm{q}, \mathrm{s}, \mathrm{t}$
(b) $\mathrm{A} \rightarrow \mathrm{r}, \mathrm{B} \rightarrow \mathrm{q}, \mathrm{r}, \mathrm{C} \rightarrow \mathrm{s}, \mathrm{D} \rightarrow \mathrm{p}, \mathrm{q}, \mathrm{s}, \mathrm{t}$
(c) $\mathrm{A} \rightarrow \mathrm{s}, \mathrm{B} \rightarrow \mathrm{t}, \mathrm{r}, \mathrm{C} \rightarrow \mathrm{q}, \mathrm{r}, \mathrm{D} \rightarrow \mathrm{p}, \mathrm{t}$
(d) $\mathrm{A} \rightarrow \mathrm{t}, \mathrm{B} \rightarrow \mathrm{q}, \mathrm{r}, \mathrm{C} \rightarrow \mathrm{r}, \mathrm{D} \rightarrow \mathrm{p}, \mathrm{t}$

Sol. $\quad \mathrm{A} \rightarrow \mathrm{r}$ (for isoelectronic species the ionic size decreases with increase in nuclear charge)
$\mathrm{B} \rightarrow \mathrm{q} \mathrm{r}$, (Number of atomic shell increases, ionic size increases)
$\mathrm{C} \rightarrow \mathrm{s}$ ( as Cl has less inter electronic repulsiums than F due to bigger size of 3P-Subshell)
$\mathrm{D} \rightarrow \mathrm{p}, \mathrm{q}, \mathrm{s}, \mathrm{t}$ ( Oxidation state increases the electronegativity increases. For isoelectronic species ionisation energy and electron affinity increasing nuclear charge)
17. If $\mathrm{K}_{1}$ and $\mathrm{K}_{2}$ are first and second ionisation constant of $\mathrm{H}_{2} \mathrm{CrO}_{4}, \mathrm{~K}_{3}$ is the ionsation constant for $\mathrm{NH}_{3}$ Column -I Column-II
(A) $0.1 \mathrm{M} \mathrm{H}_{2} \mathrm{CrO}_{4}$
(p) $p H=7+\frac{1}{2} p K_{2}-\frac{1}{2} \log c$
(B) $0.1 \mathrm{M} \mathrm{KHCrO}_{4}$
(q) $p H=7+\frac{1}{2} p K_{2}+\frac{1}{2} \log c$
(C) $0.1 \mathrm{M}\left(\mathrm{NH}_{4}\right)_{2} \mathrm{CrO}_{4}$
(r) $p H=\frac{1}{2} p K_{1}-\frac{1}{2} \log c$
(D) $0.1 \mathrm{M} \mathrm{K}_{2} \mathrm{CrO}_{4}$
(s) $p H=\frac{1}{2}\left(p K_{1}+p K_{2}\right)$
(t) $\left[\mathrm{H}^{+}\right]=\sqrt{K_{1} K_{2}}$
(a) $\mathrm{A} \rightarrow \mathrm{p}, \mathrm{B} \rightarrow \mathrm{s}, \mathrm{t}, \mathrm{C} \rightarrow \mathrm{r}, \mathrm{D} \rightarrow \mathrm{q}$
(b) $\mathrm{A} \rightarrow \mathrm{s}, \mathrm{B} \rightarrow \mathrm{p}, \mathrm{t}, \mathrm{C} \rightarrow \mathrm{q}, \mathrm{s}, \mathrm{D} \rightarrow \mathrm{p}, \mathrm{q}$
(c) $\mathrm{A} \rightarrow \mathrm{r}, \mathrm{B} \rightarrow \mathrm{s}, \mathrm{t}, \mathrm{C} \rightarrow \mathrm{p}, \mathrm{D} \rightarrow \mathrm{q}$
(d) $\mathrm{A} \rightarrow \mathrm{r}, \mathrm{B} \rightarrow \mathrm{t}, \mathrm{C} \rightarrow \mathrm{p}, \mathrm{r}, \mathrm{D} \rightarrow \mathrm{q}, \mathrm{t}$

## MATHEMATICS

## SECTION 1 (Maximum Marks: 12)

- This section contains THREE (03) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks $\quad:+4$ ONLY if (all) the correct option(s) is(are) chosen;
Partial Marks $\quad:+3$ If all the four options are correct but ONLY three options are chosen;
Partial Marks $:+2$ If three or more options are correct but ONLY two options are chosen, both of which are correct;
Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : -2 In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then choosing ONLY (A), (B) and (D) will get +4 marks; choosing ONLY (A) and (B) will get +2 marks; choosing ONLY (A) and (D) will get +2 marks; choosing ONLY (B) and (D) will get +2 marks; choosing ONLY (A) will get +1 mark; choosing ONLY (B) will get +1 mark; choosing ONLY (D) will get +1 mark; choosing no option (i.e. the question is unanswered) will get 0 marks; and choosing any other combination of options will get -2 marks.

1. The solution set of the system of equations $\log _{3} x+\log _{3} y=2+\log _{3} 2$ and $\log _{27}(x+y)=\frac{2}{3}$ is
(a) $(6,3)$
(b) $(3,6)$
(c) $(6,12)$
(d) $(12,6)$
2. If the arithmetic mean of two positive numbers $a \& b(a>b)$ is twice their geometric mean, then $a: b$ is
(a) $2+\sqrt{3}: 2-\sqrt{3}$
(b) $7+4 \sqrt{3}: 1$
(c) $1: 7-4 \sqrt{3}$
(d) $2: \sqrt{3}$
3. If $\sum_{r=1}^{n} r(r+1)(2 r+3)=a n^{4}+b n^{3}+c n^{2}+d n+e$, then
(a) $a+c=b+d$
(b) $\mathrm{e}=0$
(c) a, b-2/3, c-1 are in A.P. (d) c/a is an integer

## SECTION 2 (Maximum Marks: 12)

- This section contains FOUR (04) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.

Answer to each question will be evaluated according to the following marking scheme:
Full Marks $\quad:+3$ If ONLY the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : -1 In all other cases.
4. If $x=\sum_{n=0}^{\infty} a^{n}, y=\sum_{n=0}^{\infty} b^{n}, z=\sum_{n=0}^{\infty} c^{n}$. Where $\mathrm{a}, \mathrm{b}, \mathrm{c}$ are in AP and $|\mathrm{a}|<1,|\mathrm{~b}|<1,|\mathrm{c}|<1$, then $\mathrm{x}, \mathrm{y}, \mathrm{z}$ are in
(a) HP
(b) Arithmetic-Geometric Progression
(c) AP
(d) GP
5. Number of values ' p ' for which the equation $\left(\mathrm{p}^{2}-3 \mathrm{p}+2\right) x^{2}-\left(\mathrm{p}^{2}-5 \mathrm{p}+4\right) x+\mathrm{p}-\mathrm{p}^{2}=0$ possess more than two roots, is
(a) 0
(b) 1
(c) 2
(d) None of these
6. If $2 \sec ^{2} \alpha-\sec ^{4} \alpha-2 \operatorname{cosec}^{2} \alpha+\operatorname{cosec}^{4} \alpha=15 / 4$, then $\tan \alpha$ is equal to
(a) $1 / \sqrt{2}$
(b) $1 / 2$
(c) $\frac{1}{2 \sqrt{2}}$
(d) $1 / 4$
7. If $\tan ^{2} \theta=2 \tan ^{2} \phi+1$, then the value of $\cos 2 \theta=\sin ^{2} \phi$ is
(a) 1
(b) 2
(c) -1
(d) Independent of $\phi$

## SECTION 3 (Maximum Marks: 24)

- This section contains SIX (06) questions.
- The answer to each question is a NON-NEGATIVE INTEGER.
- For each question, enter the correct integer corresponding to the answer on OMR sheet.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks $\quad:+4$ If ONLY the correct integer is entered;
Zero Marks : 0 In all other cases.
8. The lines $x y=0$ and $x+y=17$ form a triangle in the $x-y$ plane. The total number of points having co-ordinates which are prime numbers and lie inside the triangle is

Sol. Prime number between 0 \& 17; 2, 3, 5, 7, 11, 13
With 2 as x -co-ordinate, total points $=6$
With 3 as $x$-co-ordinate, total points $=6$
With 5 as $x$-co-ordinate, total points $=5$
With 7 as $x$-co-ordinate, total points $=4$
With 11 as $x$-co-ordinate, total points $=3$


With 13 as x -co-ordinate, total points $=2$
Total points $=26$
9. If the coefficients of $x^{7} \& x^{8}$ in the expansion of $\left[2+\frac{x}{3}\right]^{n}$ are equal, then the value of n is

Sol. Coefficient of $\mathrm{T}_{8}=\mathrm{T}_{9}$
Then, ${ }^{n} C_{7} \cdot 2^{n-7} \cdot 3^{-7}={ }^{n} C_{8} 2^{n-8} \cdot 3^{-8}$

$$
\begin{aligned}
& { }^{{ }^{n} C_{7} \cdot 2 \cdot 3}={ }^{n} C_{8} \\
& \frac{6 . n!}{(n-7)!\cdot 7!}=\frac{n!}{(n-8)!.8!} \\
& 6(8)=n-7 \\
& \therefore n=55
\end{aligned}
$$

10. The value of the expression $\log _{2}\left(1+\frac{1}{2} \sum_{k=1}^{11}{ }^{12} c_{k}\right)$ is

Sol. $\quad \sum_{k=1}^{11}{ }^{12} c_{k}=\sum_{k=0}^{12}{ }^{12} c_{k}-{ }^{12} c_{0}-{ }^{12} c_{12}=2^{12}-1-1=2^{12}-2$
$\Rightarrow \log _{2}\left(1+\frac{1}{2} \sum_{k=1}^{11}{ }^{12} c_{k}\right)=\log _{2}\left(1+2^{11}-1\right)=\log _{2} 2^{11}=11$
11. The number of integer values of $m$, for which the $x$ co-ordinate of the point of intersection of the lines $3 x+4 \mathrm{y}=9$ and $\mathrm{y}=\mathrm{m} x+1$ is also an integer, is
ans :- 2
Sol.

$$
\begin{aligned}
& 3 x+4 y=9, y=m x+1 \\
& 3 x+4 m x+4=9 \\
& x=\frac{5}{(3+4 m)} 5 \text { is divisible by }(3+4 m) \\
& 4 m+3= \pm 1 \text { or } 4 m+3= \pm 5 \\
& m=-\frac{1}{2},-1 \text { or } m=\frac{1}{2},-2 \text { Two values }
\end{aligned}
$$

12. Let a and b be two nonzero real numbers. If the coefficient of $x^{5}$ in the expansion of $\left(a x^{2}+\frac{70}{27 b x}\right)^{4}$ is equal to the coefficient of $x^{-5}$ in the expansion of $\left(a x-\frac{1}{b x^{2}}\right)^{7}$ then the value of 2 b is $\qquad$ .

Sol. $\quad T_{r+1}={ }^{4} C_{r}\left(a x^{2}\right)^{4-}\left(\frac{70}{27 b x}\right)^{r}$

$$
\text { For coefficient of } x^{5}, 8-2 r-r=5 \Rightarrow r=1
$$

$\Rightarrow$ Coefficient of $x^{5}={ }^{4} C_{1} a^{3}\left(\frac{70}{27 b}\right)$
$t_{r+1}={ }^{7} C_{r}(a x)^{7-r}\left(-\frac{1}{b x^{2}}\right)^{r}$
For coefficient of $x^{-5}, 7-r-2 r=-5 \Rightarrow r=4$
$\Rightarrow$ coefficient of $x^{-5}={ }^{7} C_{4} a^{3} \frac{1}{b^{4}}$
$\Rightarrow{ }^{4} C_{1} a^{3}\left(\frac{70}{27 b}\right)={ }^{7} C_{4} a^{3} \frac{1}{b^{4}} \Rightarrow 2 b=3$
13. The length of the latus rectum of the ellipse $2 x^{2}+3 y^{2}-4 x-6 y-13=0$ is

Sol.
ans:-4

## SECTION 4 (Maximum Marks: 12)

- This section contains FOUR (04) Matching List Sets.
- Each set has ONE Multiple Choice Question.
- Each set has TWO lists: List-I and List-II.
- List-I has Four entries (P), (Q), (R) and (S) and List-II has Five entries (1), (2), (3), (4) and (5).
- FOUR options are given in each Multiple Choice Question based on List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks $\quad:+3$ ONLY if the option corresponding to the correct combination is chosen;
Zero Marks $\quad: \quad 0$ If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : -1 In all other cases.
14. Column A
(A) $\frac{x^{2}-5 x+6}{x^{2}-4 x+3} \geq 0$

Column B
(B) $\frac{x-2}{x-1} \geq 0$
(q) $(-\infty, 1] \cup[2,3) \cup(3, \infty)$
(C) $\frac{(x-2)(x-1)}{(x-3)^{2}} \geq 0$
(r) $(-\infty, 1) \cup[2,3) \cup(3, \infty)$
(D) $(x-1)(x-2)(x-3) \geq 0$
(s) $(-\infty, 1) \cup[2, \infty)$
(a) $\mathrm{A} \rightarrow \mathrm{s}, \mathrm{B} \rightarrow \mathrm{s}, \mathrm{C} \rightarrow \mathrm{r}, \mathrm{D} \rightarrow \mathrm{p}$
(b) $\mathrm{A} \rightarrow \mathrm{r}, \mathrm{B} \rightarrow \mathrm{s}, \mathrm{C} \rightarrow \mathrm{q}, \mathrm{D} \rightarrow \mathrm{p}$
(c) $\mathrm{A} \rightarrow \mathrm{s}, \mathrm{B} \rightarrow \mathrm{s}, \mathrm{C} \rightarrow \mathrm{q}, \mathrm{D} \rightarrow \mathrm{q}$
(d) $\mathrm{A} \rightarrow \mathrm{r}, \mathrm{B} \rightarrow \mathrm{s}, \mathrm{C} \rightarrow \mathrm{r}, \mathrm{D} \rightarrow \mathrm{q}$
15. Column A
(A) Average of first $n$ whole number
(B) Average of first $n$ odd natural number
(C) Average of the oveservations 1, 3, 5, 7....,n
(D) Average of first $n$ even natural number

## Column B

(p) $n$
(q) $n+1$
(r) $\frac{n-1}{2}$
(a) $\mathrm{A} \rightarrow \mathrm{r}, \mathrm{B} \rightarrow \mathrm{r}, \mathrm{C} \rightarrow \mathrm{p}, \mathrm{D} \rightarrow \mathrm{q}$
(b) $\mathrm{A} \rightarrow \mathrm{r}, \mathrm{B} \rightarrow \mathrm{p}, \mathrm{C} \rightarrow \mathrm{q}, \mathrm{D} \rightarrow \mathrm{s}$
(c) $\mathrm{A} \rightarrow \mathrm{r}, \mathrm{B} \rightarrow \mathrm{p}, \mathrm{C} \rightarrow \mathrm{s}, \mathrm{D} \rightarrow \mathrm{q}$
(d) $\mathrm{A} \rightarrow \mathrm{s}, \mathrm{B} \rightarrow \mathrm{r}, \mathrm{C} \rightarrow \mathrm{q}, \mathrm{D} \rightarrow \mathrm{p}$
16. From point $\mathrm{P}(12,0)$ three normal $\mathrm{PA}, \mathrm{PB}$ and PC are drawn to the parabola $\mathrm{y}^{2}=16 x$ where $\mathrm{A}, \mathrm{B}, \mathrm{C}$ lying on the parabola

Column A

## Column B

(A) Area of $\triangle \mathrm{ABC}$
(p) 32
(B) If centroid of $\triangle \mathrm{ABC}$ is $(x, y)$ then $9 x+7 y$ is equal to
(q) 24
(C) If circumcenter of $\triangle \mathrm{ABC}$ is $(\alpha, \beta)$ then $2 \alpha+\beta$ is equal to
(r) 16
(D) Circum-radius of $\triangle \mathrm{ABC}$ is
(s) 20
(t) 10
(a) $\mathrm{A} \rightarrow \mathrm{p}, \mathrm{B} \rightarrow \mathrm{q}, \mathrm{C} \rightarrow \mathrm{s}, \mathrm{D} \rightarrow \mathrm{t}$
(b) $\mathrm{A} \rightarrow \mathrm{t}, \mathrm{B} \rightarrow \mathrm{s}, \mathrm{C} \rightarrow \mathrm{q}, \mathrm{D} \rightarrow \mathrm{r}$
(c) $\mathrm{A} \rightarrow \mathrm{r}, \mathrm{B} \rightarrow \mathrm{s}, \mathrm{C} \rightarrow \mathrm{t}, \mathrm{D} \rightarrow \mathrm{p}$
(d) $\mathrm{A} \rightarrow \mathrm{p}, \mathrm{B} \rightarrow \mathrm{q}, \mathrm{C} \rightarrow \mathrm{r}, \mathrm{D} \rightarrow \mathrm{t}$
17. ColumnA

Column B
(A) Maximum value of $y=\frac{1-\tan ^{2}(\pi / 4-x)}{1+\tan ^{2}(\pi / 4-x)}$
(p) 1
(B) Minimum value of $\log _{3}\left(\frac{5 \sin x-12 \cos x+26}{13}\right)$
(q) 0
(C) Minimum value of $y=-2 \sin ^{2} x+\cos x+3$
(r) $\frac{7}{8}$
(D) Maximum value of $y=4 \sin ^{2} \theta+4 \sin \theta \cos \theta+\cos ^{2} \theta$
(s) 5
(t) 6
(a) $\mathrm{A} \rightarrow \mathrm{p}, \mathrm{B} \rightarrow \mathrm{q}, \mathrm{C} \rightarrow \mathrm{s}, \mathrm{D} \rightarrow \mathrm{r}$
(b) $\mathrm{A} \rightarrow \mathrm{p}, \mathrm{B} \rightarrow \mathrm{q}, \mathrm{C} \rightarrow \mathrm{r}, \mathrm{D} \rightarrow \mathrm{s}$
(c) $\mathrm{A} \rightarrow \mathrm{p}, \mathrm{B} \rightarrow \mathrm{q}, \mathrm{C} \rightarrow \mathrm{s}, \mathrm{D} \rightarrow \mathrm{r}$
(d) $\mathrm{A} \rightarrow \mathrm{q}, \mathrm{B} \rightarrow \mathrm{p}, \mathrm{C} \rightarrow \mathrm{s}, \mathrm{D} \rightarrow \mathrm{r}$

