Solution

Section - A Physics

1. A pan is going down along y-axis with it's position changing with time as $y(t) = \sqrt{2}(\sin\omega t + \cos\omega t)$. A mass M = 5kg is placed on the pan at t = 0 . Find the minimum value of 'ω_{min}' for which mass leaves the contact with the pan and also calculate the time 't' for which this happen for the first time. (Take g = 10 m/s^2)

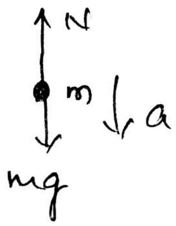
(a) $\omega_{min} = 1$ rad/sec & $t = \frac{22}{28}$ sec (b) $\omega_{min} = \frac{22}{28}$ & $t = 1$ sec

(c) $\omega_{min} = \sqrt{5}$ & $t = \frac{22}{28\sqrt{5}}$ sec (d) $\omega_{min} = \frac{22}{28\sqrt{5}}$ & $t = \sqrt{5}$ sec

Sol. $y(t) = \sqrt{2}(\sin\omega t + \cos\omega t)$. $\Rightarrow y(t) = 2\left[\frac{1}{\sqrt{2}}\sin\omega t + \frac{1}{\sqrt{2}}\cos\omega t\right] \Rightarrow y(t) = 2\sin\left(\omega t + \frac{\pi}{4}\right)$

$$a(t) = -2\omega^2 \sin\left(\omega t + \frac{\pi}{4}\right)$$

F.B.D. of mass :-



mg-N = ma

$N \to 0 \Rightarrow g = a(t)$

$$-10 = -2\omega^2 \sin\left(\omega t + \frac{\pi}{4}\right) \Rightarrow \omega = \sqrt{\frac{5}{\sin\left(\omega t + \frac{\pi}{4}\right)}}$$

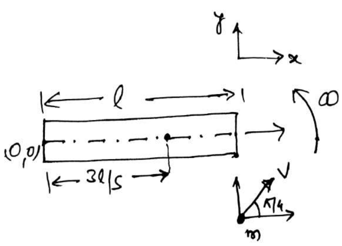
for ω to be minimum $\sin\left(\omega t + \frac{\pi}{4}\right) = 1 \Rightarrow \omega_{min} = \sqrt{5}\ rad/\sec$

$$\omega_{min} t + \frac{\pi}{4} = \frac{\pi}{2} \Rightarrow t = \frac{22}{7\times4\times\sqrt{5}} \sec$$

2. A uniform rod of mass m = 4m and length ℓ is kept on a frictionless surface in xy plane such that one end of the rod is at origin and its principle axis is coinciding with x-axis. A mass m moving with velocity v making an angle $\theta = \frac{\pi}{4}$ with the principle axis of the rod collides with one end of the rod and sticks to it. The angular speed of the rod + mass system just after the collision is

(a) $\frac{3v}{7\sqrt{2}\ell}$ (b) $\frac{3\sqrt{2}v}{7\ell}$ **(c)** $\frac{3v}{4\sqrt{2}\ell}$ (d) $\frac{3v}{7\ell}$

Sol. Since inelastic collision has happened the COM will shift therefore angular momuntum will be conserved about new COM.



$$\frac{4m\left(\frac{\ell}{2}\right) + m\ell}{5m} = \frac{3\ell}{5}$$

Angular momentum before collision about this COM

$$\left|\vec{L}_k\right| = mv\sin\frac{\pi}{4}\times\frac{2\ell}{5}$$

and angular momentum after collision = $I\omega$

$$=\left(\frac{4m\ell^2}{12}+4m\left(\frac{\ell}{10}\right)^2+m\times\frac{4\ell^2}{25}\right)\omega \Rightarrow \frac{2m\ell v}{5\sqrt{2}}=\frac{8}{15}\omega m\ell^2 \Rightarrow \omega=\frac{3v}{4\sqrt{2}\ell}$$

3. Consider a solid sphere of radius R and mass density $\rho(r)=\rho_0\left(1-\frac{r^2}{R^2}\right)$, $0 < r \le R$. The minimum density of a liquid in which it will float is

(a) $\frac{\rho_0}{5}$ (b) $\frac{2\rho_0}{5}$ (c) $\frac{2\rho_0}{3}$ (d) $\frac{\rho_0}{3}$

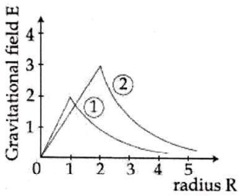
Sol. $\rho=\rho_0\left(1-\frac{r^2}{R^2}\right)$ $0 < r \le R$

mg = B

$\int\rho(4\pi r^2 dr)=\rho_L\frac{4}{3}\pi R^3$; $\int_0^R\rho_0\left(1-\frac{r^2}{R^2}\right)4\pi r^2 dr=\rho_L\frac{4}{3}\pi R^3$

$\int_0^R\rho_0 4\pi\left(r^2-\frac{r^4}{R^2}\right)dr=\rho_0 4\pi\left(\frac{r^3}{3}-\frac{r^5}{5R^2}\right)_0^R=\rho_L\frac{4}{3}\pi R^3$; $\frac{2}{5}\rho_0=\rho_L$

4. Consider two solid spheres of radii R_1 = 1m, R_2 = 2m and masses M_1 and M_2, respectively. The gravitational field due to sphere (1) and (2) are shown. The value of $\frac{M_1}{M_2}$ is:



(a)

Sol. C

$3=\frac{Gm_2}{2^2}$; $2=\frac{Gm_1}{1^2}$

$\therefore \frac{3}{2}=\frac{1}{4}\frac{m_2}{m_1}$

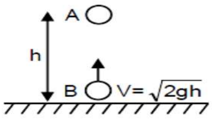
$\frac{m_1}{m_2}=\frac{1}{6}$

5. A particle of mass m is dropped from a height h above the ground. At the same time another particle of same mass is thrown vertically upwards from the ground with a speed of $\sqrt{2gh}$. If they collide head-on completely inelastically, the time taken for the combined mass to reach the ground, in units of $\sqrt{\frac{h}{g}}$ is :

(a) $\frac{1}{2}$ (b) $\sqrt{\frac{1}{2}}$ (c) $\sqrt{\frac{3}{4}}$ (d) $\sqrt{\frac{3}{2}}$

Sol. D

Time for collision $t_1 = \frac{h}{\sqrt{2gh}}$



After t_1

$V_A = 0 - gt_1 = -\sqrt{\frac{gh}{2}}$

and $V_B = \sqrt{2gh} - gt_1 = \sqrt{gh}\left[\sqrt{2} - \frac{1}{\sqrt{2}}\right]$

at the time f collision

$\vec{P}_i = \vec{P}_f$

$\Rightarrow m\vec{V}_A + m\vec{V}_B = 2m\vec{V}_f$

$\Rightarrow -\sqrt{\frac{gh}{2}} + \sqrt{gh}\left[\sqrt{2} - \frac{1}{\sqrt{2}}\right] = 2\vec{V}_f$

$V_f = 0$

and height from ground $= h - \frac{1}{2}gt_1^2 = h - \frac{h}{4} = \frac{3h}{4}$

So time $= \sqrt{2 \times \frac{\left(\frac{3h}{4}\right)}{g}} = \sqrt{\frac{3h}{2g}}$

6. Sol. A

$\vec{r} = \cos\omega t\hat{i} + \sin\omega t\hat{j}$

$\vec{v} = \frac{d\vec{r}}{dt} = \omega(-\sin\omega t\hat{i} + \cos\omega t\hat{j})$

$\vec{a} = \frac{d\vec{v}}{dt} = -\omega^2(\cos\omega t\hat{i} + \sin\omega t\hat{j})$

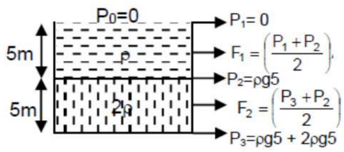
$\vec{a} = -\omega^2\vec{r}$ $\quad\therefore\quad$ $\vec{a}$ is antiparallel to $\vec{r}$

$\vec{v}\cdot\vec{r} = \omega(-\sin\omega t\cos\omega t + \cos\omega t\sin\omega t) = 0$

So, $\vec{V} \perp \vec{r}$

7. Sol. B

$\frac{F_1}{F_2} = \frac{1}{4}$



8. Sol. B

$M_1 = \frac{4}{3}\pi R^3\rho \quad ; \quad M_2 = \frac{4}{3}\pi(1)^3\,(-\rho)$

$X_{com} = \frac{M_1X_1 + M_2X_2}{M_1 + M_2}$

$\Rightarrow \frac{\left[\frac{4}{3}\pi R^3\rho\right]0 + \left[\frac{4}{3}\pi(1)^3(-\rho)\right][R-1]}{\frac{4}{3}\pi R^3\rho + \frac{4}{3}\pi(1)^3(-\rho)} = -(2-R)$

$\Rightarrow \frac{(R-1)}{(R^3-1)} = (2-R) \qquad (R \neq 1)$

$\frac{(R-1)}{(R-1)(R^2+R+1)} = 2-R$

$(R^2 + R + 1)(2 - R) = 1$

9. Sol. If μ is Poisson's ratio,

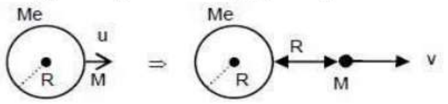
$Y = 3K(1 - 2\mu)$ ………….(1)

and $Y = 2\eta(1+\mu)$………. (2)

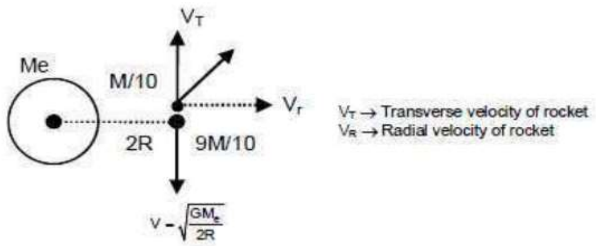
With the help of equations (1) and (2) , we can write

$\frac{3}{Y} = \frac{1}{\eta} + \frac{1}{3k} \Rightarrow K = \frac{\eta Y}{9\eta - 3Y}$

10. Sol. $\frac{-GM_eM}{R}+\frac{1}{2}Mu^2=\frac{-GM_eM}{2R}+\frac{1}{2}Mv^2$



$v=\sqrt{u^2-\frac{GM_e}{R}}$



$\frac{M}{10}V_T=\frac{9M}{10}\sqrt{\frac{GM_e}{2R}}$; $\frac{M}{10}V_r=M\sqrt{u^2-\frac{GM_e}{R}}$

Kinetic energy = $\frac{1}{2}\frac{M}{10}\left(V_T^2+V_r^2\right)=\frac{M}{20}\left(81\frac{GM_e}{2R}+100\,u^2-100\frac{GM_e}{R}\right)$

$=\frac{M}{20}\left(100\,u^2-\frac{119GM_e}{2R}\right)$

$=5M\left(u^2-\frac{119GM_e}{200R}\right)$

11. Sol. Stress = Y× strain

$\Rightarrow \quad \frac{T_1}{A}=Y\times-\frac{(\ell_1-\ell)}{\ell} \quad$ (i)

$\frac{T_2}{A}=Y\times\frac{(\ell_2-\ell)}{\ell} \quad$ (ii)

$\Rightarrow \quad \frac{T_1}{T_2}=\frac{\ell_1-\ell}{\ell_2-\ell}$

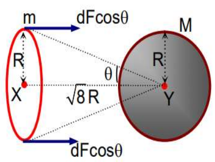
$\Rightarrow \ell=\frac{T_1\ell_2-T_2\ell_1}{T_1-T_2}=\frac{T_2\ell_1-T_1\ell_2}{T_2-T_1}$

12. Sol. **Method-I:**

$F=M_{sphere}\,E_{ring}=M\times\frac{Gmx}{\left(R^2+x^2\right)^{3/2}}=M\times\frac{Gmx\sqrt{8}R}{\left(R^2+8R^2\right)^{3/2}}=\frac{\sqrt{8}\,GMm}{27R^2}$

Method-II:

$F=\int dF\cos\theta=\cos\theta\int(dm)\frac{GM}{\left(R^2+8R^2\right)}=\frac{\sqrt{8}GMm}{27R^2}$



13. Sol. $F = \frac{k}{R^3} = \frac{mv^2}{R} \Rightarrow v = \sqrt{\frac{k}{mR^2}} = \sqrt{\frac{k}{m}} \times \frac{1}{R}$

$$T = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{\frac{k}{m}} \times \left(\frac{1}{R}\right)} \Rightarrow T \propto R^2$$

14. Sol. mass m will acquire velocity 2u. Total momentum of system will be conserved but total kinetic energy is conserved during elastic collision

15. Sol. From volume conservation

$$n \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 \Rightarrow R^3 = nr^3 \text{ .... } (1)$$

Decrease in surface area $= n \times 4\pi r^2 - 4\pi R^2$

$$(\Delta A) = 4\pi[nr^2 - R^2] = 4\pi\left[\frac{n \times r^3}{r} - R^2\right] = 4\pi\left[\frac{R^3}{r} - R^2\right] = 4\pi R^3\left[\frac{1}{r} - \frac{1}{R}\right]$$

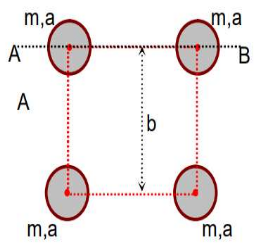
Energy released (W)= $T \times \Delta A = 4\pi R^3 T\left[\frac{1}{r} - \frac{1}{R}\right]$

Heat produced (Q) $= \frac{W}{J} = \frac{4\pi T R^3}{J}\left[\frac{1}{r} - \frac{1}{R}\right]$

Now, $Q = ms\,\Delta\theta$

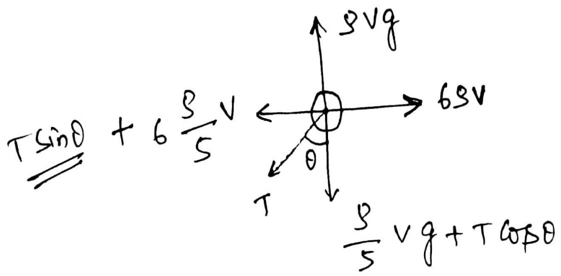
$$\Rightarrow \frac{4\pi R^3 T}{J}\left[\frac{1}{r} - \frac{1}{R}\right] = \left(\frac{4}{3}\pi R^3\right) \times |x| \times \Delta\theta \Rightarrow \Delta\theta = \frac{3T}{J}\left[\frac{1}{r} - \frac{1}{R}\right]$$

16. Sol. $I_{AB} = \frac{2}{5}ma^2 \times 2 + \left(\frac{2}{5}ma^2 + mb^2\right) \times 2 = \frac{8ma^2}{5} + 2mb^2$



17. Sol. In elliptical orbit, areal velocity is constant

18. Sol.



$T\sin\theta = \frac{24}{5}\rho V$

$T\cos\theta = \frac{4\rho Vg}{5}$

$\rightarrow \tan\theta = \frac{6}{10}$

$\Rightarrow \boxed{\theta = \tan^{-1}\frac{3}{5}}$

19. Sol. C

$$2\times\frac{1}{2g}\left(\frac{p\sin\theta}{m}\right)^2=\frac{2p\sin\theta}{mg}\times\frac{p\cos\theta}{m}$$

$$\frac{1}{2}\sin^2\theta=\sin\theta\cos\theta\Rightarrow\tan\theta=2$$

$$\therefore\ \cos\theta=\frac{1}{\sqrt{5}}$$

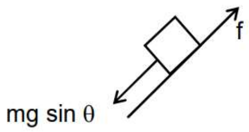
$$\text{Minimum kinetic energy}=\frac{(p\cos\theta)^2}{2m}=\frac{p^2}{2m}\times\frac{1}{5}=\frac{p^2}{10m}$$

20.

(a) $\frac{3\rho agt}{5v}$ (b) $\frac{4\rho agt}{5v}$ (c) $\frac{\rho agt}{v}$ (d) $\frac{5\rho agt}{3v}$

Sol. A

$$mg\sin\theta=\eta A\frac{v}{t}$$



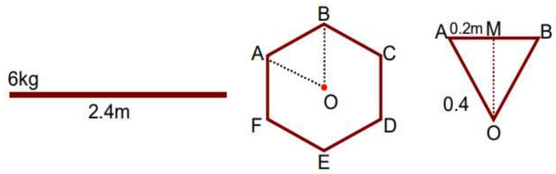
$$a^3\rho g\times\frac{3}{5}=\eta\times a^2\times\frac{v}{t}$$

$$\frac{3}{5}a\rho g=\frac{\eta v}{t}$$

$$\eta=\frac{3\rho agt}{5v}$$

Section - B Physics

1. Sol.



Let $m_{AB}=m=1\text{kg}$

$AB=0.4\text{m}=\ell$

$d=OM=\sqrt{0.16-0.04}=\sqrt{0.12}$

$d=2\sqrt{3}\times10^{-1}$

$I_{AB\text{ about }O}=\frac{m\ell^2}{12}+md^2$

$\therefore I_{\text{hexagon, about }O}=6\left[\frac{m\ell^2}{12}+md^2\right]=\frac{m\ell^2}{2}+6md^2$

$=\frac{1\times(0.4)^2}{2}+6\times1\times\left(2\sqrt{3}\times10^{-1}\right)^2=\frac{0.16}{2}+6\times4\times3\times10^{-2}$

$=0.08+0.72=0.8\text{kgm}^2=8\times10^{-1}\text{kgm}^2$

2. Sol. For equilibrium ,

$$\frac{dU}{dr}=0\Rightarrow\frac{-10\alpha}{r^{11}}+\frac{5\beta}{r^6}=0\Rightarrow r=\left(\frac{2\alpha}{\beta}\right)^{1/5}$$

3. Sol. By $A_1V_1=A_2V_2$

$\Rightarrow\ \pi(20)^2\times5=\pi(1)^2V_2\quad\Rightarrow V_2=2\,\text{m/s}^2$

4. Sol. Using conservation of energy :

$$\Rightarrow \frac{1}{2}mv_i^2 - \frac{Gmm}{R} = 0 - \frac{Gmm}{11R}$$

$$\Rightarrow v_i^2 = \frac{20}{11}\frac{Gm}{R}$$

Escape velocity is defined as $\sqrt{\frac{2GM}{R}}$

$$v_i = \sqrt{\frac{10}{11}}v_e$$

5. Sol. If angle between them is 180^0 then

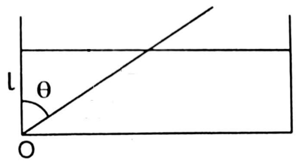
$$\vec{P} \times \vec{Q} = \vec{q} \times \vec{P} = 0$$

6. Sol. $$\frac{GM}{(3R/2)^2} = \frac{GM}{R^3} \times r$$

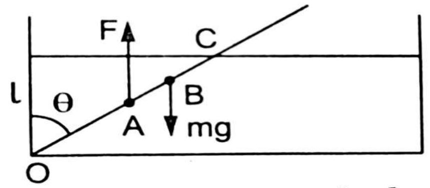
$$\Rightarrow OA = \frac{4R}{9} = r$$

$$AB = R - \frac{4R}{9} = \frac{5R}{9} \Rightarrow OA : AB = 4 : 5 = x : y \Rightarrow x = 4$$

7. A wooden plank of length 1 m and uniform cross section is hinged at one end to the bottom of a tank as shown in figure . The tank is filled with water up to a height of 0.5 m. The specific gravity of the plank is 0.5. Find the angle θ that the plank makes with the vertical in the equilibrium position. (Exclude the case $\theta = 0$.)



Sol.



The forces acting on the plank are shown in the figure. The height of water level is $l = 0.5$ m. The length of the plank is 1.0 m = $2l$. The weight of the plank acts through the centre B of the plank. We have $OB = l$. The buoyant force F acts through the point A which is the middle point of the dipped part OC of the plank.

We have $OA = \frac{OC}{2} = \frac{l}{2\cos\theta}$.

Let the mass per unit length of the plank be ρ.

Its weight $mg = 2l\rho g$.

The mass of the part OC of the plank $= \left(\frac{l}{\cos\theta}\right)\rho$.

The mass of water displaced $= \frac{1}{0.5}\frac{l}{\cos\theta}\rho = \frac{2l\rho}{\cos\theta}$.

The buoyant force F is, therefore, $F = \frac{2l\rho g}{\cos\theta}$.

Now, for equilibrium, the torque of mg about O should balance the torque of F about O.

So, $mg(OB)\sin\theta = F(OA)\sin\theta$

or, $(2l\rho)l = \left(\frac{2l\rho}{\cos\theta}\right)\left(\frac{l}{2\cos\theta}\right)$

or, $\cos^2\theta = \frac{1}{2}$

or, $\cos\theta = \frac{1}{\sqrt{2}}$, or, $\theta = 45°$.

8. A large wooden plate of area 10 m^2 floating on the surface of a river is made to move horizontally with a speed of 2 m s^{-1} by applying a tangential force. If the river is 1m deep and the water in contact with the bed is stationary. If F be the tangential force needed to keep the plate moving. Calculate 100×F = ____N. Coefficient of viscosity of water at the temperature of the river = 10^{-3} poise.

Sol. The velocity decreases from 2 m s^{-1} to zero in 1 m of perpendicular length. Hence, velocity gradient

$$= dv/dx = 2\ s^{-1}.$$

Now, $$\eta = \left|\frac{F/A}{dv/dx}\right|$$

or, $$10^{-3}\ \frac{N\text{–}s}{m^2} = \frac{F}{(10\ m^2)(2\ s^{-1})}$$

or, $$F = 0{\cdot}02\ N.$$

9. Sol. $y = \frac{\text{stress}}{\text{strain}}$

$\Rightarrow \text{strain} = \frac{\text{stress}}{y} = \frac{F}{Ay}$

$= \frac{62.8\times1000}{\pi r^2\times2\times10^{11}} = \frac{62.8\times1000}{3.14\times400\times10^{-6}\times2\times10^{11}}$

$= \frac{200}{8}\times10^{-5} = 25\times10^{-5}$

10. Sol. At steady state :-

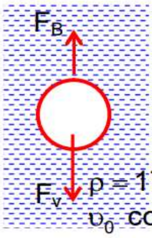
$F_B = F_V$

$\Rightarrow \frac{4}{3}\pi r^3\rho g = 6\pi n r v$

$\Rightarrow n = \frac{4\pi r^3\rho g}{18\pi r v} = \frac{2r^2\rho g}{9v}$

$= \frac{2}{9}\times\frac{9\times10^{-6}\times1750\times10}{35\times10^{-4}}$

$= 10$



$\rho = 1750\,ky/m^3$

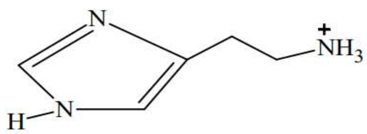
υ_0 coeff of vicosy, n = ?

radius r = 3mm

Chemistry

1. Sol **(B)**

2. Sol.



The N-atoms present in the ring will have same pK_a values (6.0), while N atom outside the ring will have different pK_a value ($pK_a > 7.4$)

Therefore, two N-atoms inside the ring will remain in unprotonated form in human blood because their $pK_a(6.0) <$ pH of blood (7.4), while the N-atom outside the ring will remain in protonated form because its $pK_a >$ pH of blood (7.4).

3. Sol. $\underset{450\text{ mL}}{Ba^{+2}} + \underset{50\text{ mL},1M}{Na_2SO_4} \longrightarrow BaSO_4 + 2Na^+$

$K_{sp}(BaSO_4) = [Ba^{+2}][SO_4^{-2}]$

$10^{-10} = [Ba^{+2}] \times 0.1$

$[Ba^{+2}] = 10^{-9}$ M (in 500 mL solution)

$[SO_4^{-2}]$ in 500 mL solution will be

$50 \times 1 = M \times 500$

$M = 0.1$

Now, we have to calculate $[Ba^{+2}]$ in original solution (450 mL)

$10^{-9} \times 500 = 450 \times M$

$$M = \frac{10^{-9} \times 500}{450} = \frac{10}{9} \times 10^{-9} = 1.11 \times 10^{-9} \text{ M}$$

4. Sol. Ratio of mass % of C and H in $C_xH_yO_z$ is 6 : 1.
Therefore,
Ratio of mole % of C and H in $C_xH_yO_z$ will be 1 : 2.
Therefore x : y = 1 : 2, which is possible in options 1, 2 and 3.
Now oxygen required to burn C_xH_y

$$C_xH_y + \left(x + \frac{y}{4}\right)O_2 \longrightarrow xCO_2 + \frac{y}{2}H_2O$$

Now z is half of oxygen atoms required to burn C_xH_y.

$$\therefore z = \frac{\left(2x + \frac{y}{2}\right)}{2} = \left(x + \frac{y}{4}\right)$$

Now putting values of x and y from the given options:
Option (1), x = 2, y = 4

$$z = \left(2 + \frac{4}{4}\right) = 3$$

Option 2, x = 3, y = 6

$$z = \left(3 + \frac{6}{4}\right) = 4.5$$

Therefore correct option is 1 ($C_2H_4O_3$)

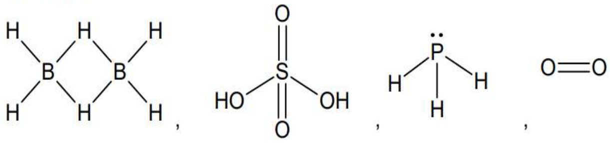
5. Which of the following compound contain only super primary carbon atom ?
(A) Dimethyl ether (B) Ethyl methyl ether (C) Acetaldehyde (D) Acetone

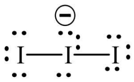
6. Sol. Both BCl_3 and $AlCl_3$ are Lewis acids as both 'B' and 'Al' has vacant p-orbitals. $SiCl_4$ is also a Lewis acid as silicon atom has vacant 3d-orbital.

7. Sol.

Species	Bond order
(1) H_2^{2-}	B. O. $= \frac{1}{2}[2-2] = 0$ (does not exist)
(2) He_2^{2+}	B. O. $= \frac{1}{2}[2-0] = 1$ (exists)
(3) He_2^{+}	B. O. $= \frac{1}{2}[2-1] = 0.5$ (exists)
(4) H_2^{-}	B. O. $= \frac{1}{2}[2-1] = 0.5$ (exists)

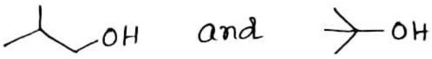
8. Sol. KCl contains only ionic bond between K^+ and Cl^- ions (K^+Cl^-) while other structures have covalent bonds as follows:



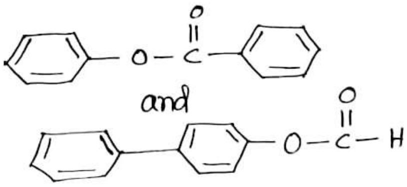
9. Sol. 

10. Which of the following compounds are not positional isomers ?

(A) $H_3C-\underset{CH_3}{CH}-\underset{CH_3}{CH}-CH_2-CH_3$ and $H_3C-\underset{CH_3}{\overset{CH_3}{C}}-CH_2-CH_2-CH_3$

(B) 

(C) 

(D) 

11. Sol. $r_n = \frac{0.53n^2}{Z}\mathring{A}$

$n = 2$

$Z = 1$

$r_2 = 0.53 \times 4\mathring{A} = 2.12\mathring{A}$

12. Sol. O^{2-}, F^-, Na^+ and Mg^{2+}, all have 10 electrons each.

13. Sol. $pH = 7 + \frac{pK_a}{2} - \frac{pK_b}{2}$

$= 7 + \frac{3.2}{2} - \frac{3.4}{2}$

$= 6.9$

14. Sol. Moles of $CoCl_3.6H_2O \longrightarrow 100\text{ mL} \times 0.1\text{M} = 10 \times 10^{-3}$ moles

Ions $\rightarrow 6.023 \times 10^{23} \times 0.01$

$= 6.023 \times 10^{21}$ ions

Precipitated ions $= 1.2 \times 10^{22}$

$\therefore$ 1 Ag^+ ion and 1 Cl^- ion.

15. Among the following the dissociation constant is highest for

(A) C_6H_5OH (B) $C_6H_5CH_2OH$ (C) $CH_3C \equiv CH$ (D) $CH_3NH_3^+Cl^-$

Sol. **(d) Dissociation of proton from $CH_3 - NH_3\ Cl^-$ is very difficult due to –I effect of Cl^- and N^+ while in C_6H_5OH due to resonance stabilization of phenoxide ion proton eliminates easily, similarly due to H-bonding in $C_6H_5CH_2OH$ it can be eliminated and $CH_3C \equiv CH$ shows acidic character by triple bond by which proton can be dissociated.**

16. Sol. Total hydrogen $({}_1H^1) = \frac{10}{100} \times 75 = 7.5$ kg

If it is replaced by ${}_1H^2$ then mass will be doubled so now hydrogen mass = 15 kg

So, mass of person will be increased by 7.5 kg.

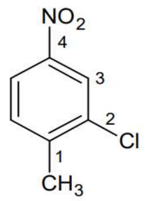
17. $M_2CO_3 + 2HCl \longrightarrow CO_2 + 2MCl + H_2O$

Moles of M_2CO_3 = Moles of CO_2 produced.

$$\text{moles of } M_2CO_3 = \frac{w}{\text{molar mass}} = 0.01186$$

$\therefore$ Molar mass = 84.3 g mol^{-1}

So, option (1) is correct.

18. 

19. $ZnO + Na_2O \longrightarrow Na_2ZnO_2$; ZnO behaving as an acid.

$ZnO + CO_2 \longrightarrow ZnCO_3$; ZnO behaving as a base.

20. An alkali is titrated against an acid with methyl orange as indicator, which of the following is a correct combination?

	Base	Acid	End point
(1)	Strong	Strong	Pink to colourless
	Base	**Acid**	**End point**
(2)	Weak	Strong	Colourless to pink
	Base	**Acid**	**End point**
(3)	Strong	Strong	Pinkish red to yellow
	Base	**Acid**	**End point**
(4)	Weak	Strong	Yellow to pinkish red

Sol. Methyl orange is weak organic base. It is used in the titration of WB vs SA

$$\underset{\substack{\text{Unionized form} \\ \text{(Yellow)}}}{MeOH} \rightleftharpoons \underset{\substack{\text{Ionized form} \\ \text{(Red)}}}{Me^+} + OH^-$$

In basic medium, equilibrium lies in backward direction and therefore it shows yellow colour.

In acidic medium, equilibrium shifts in forward direction and therefore, colour changes from yellow to red.

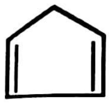
21. Sol. $AB_2 \rightleftharpoons \underset{s}{A^{2+}} + \underset{2s}{2B^-}$

$K_{sp} = 4s^3 = 3.20 \times 10^{-11}$

So, solubility $= 2 \times 10^{-4}$ mol L^{-1}

22. Sol. According to IUPAC convention for naming of elements with atomic number more than 100, different digits are written in order and at the end ium is added. For digits following naming is used.

0-nil
1-un
2-bi
3-tri
and so on…

23.  $\xrightarrow[D_2O\,(Solvent)]{Na\,(excess)}$ P Number of replaceable hydrogen by isotopes = x.

ans :- 6

24. Sol. $A_3B_{2(s)} \rightleftharpoons 3A^{+2}_{(aq)} + 2B^{3-}_{(aq)}$

Solubility $\frac{x}{M}$ $\quad \frac{3x}{M}$ $\quad \frac{x}{M}$

$\therefore Ksp = [A^{+2}]^3[B^{-3}]^2$

$= \left(\frac{3x}{M}\right)^3\left(\frac{2x}{M}\right)^2$

$= 27\left(\frac{x}{M}\right)^3 \times 4\left(\frac{x}{M}\right)^2$

$= 108\left(\frac{x}{M}\right)^5$

$Ksp = a\left(\frac{x}{M}\right)^5 = 108\left(\frac{x}{M}\right)^5$

Ans. $\therefore a = 108$

25. The total number of cyclic isomers possible for a hydrocarbon with the molecular formula C_4H_6 is __5___.

26. Sol. Molarity (M) = $\frac{w \times 1000}{\text{molecular mass} \times \text{volume of solution (ml)}}$

$= \frac{6.3 \times 1000}{126 \times 250} = \frac{4}{20} = 0.2M$

Molecular mass of oxalic acid $(H_2C_2O_4.2H_2O)$

$= 1 \times 2 + 12 \times 2 + 16 \times 4 + 2 \times 18$

$= 26 + 64 + 36 = 126$

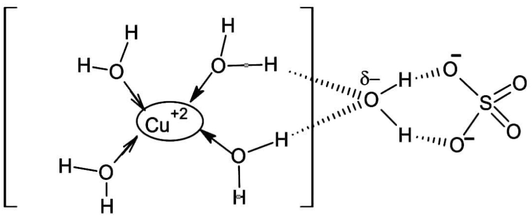
$M = 2 \times 10^{-1}M$

$= 20 \times 10^{-2}M$

$\therefore x \times 10^{-2} = 20 \times 10^{-2}$

$\therefore x = 20$

Ans.= 20

27. Sol. 

One water molecule is associated with hydrogen bond.

Ans.= 1

28. Sol. Ge (Z = 32)
$= 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^2\ 3d^{10}\ 4p^2$
$m_\ell = 0$ (for 4s, 3s, 2s, 1s) 4 orbital
$m_\ell = 0$ (one p orbital of 2p and 3p)
$m_\ell = 0$ (one d orbital)
Total orbitals = 7
Ans. = 7

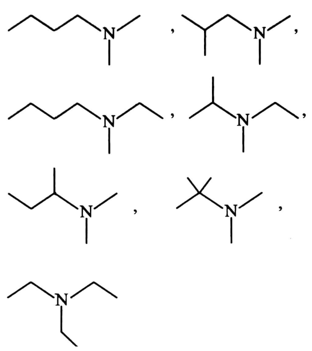
29. Sol. $PbI_2 \rightleftharpoons Pb^{2+} + 2I^-$ $\quad K_{sp} = 8.0 \times 10^{-9}$

$Pb(NO_3)_2 \rightarrow Pb^{2+} + 2NO_3^-$
0.1M $\quad$ 0.1M $\quad$ 0.1M

$PbI_2 \rightleftharpoons Pb^{2+} + 2I^-$
S $\quad$ 2S
(S+0.1)

$K_{sp} = 8\times10^{-9}$; Using $K_{sp} = [Pb^{2+}][I^-]^2$
$8\times10^{-9} = 0.1 \times (2S)^2$ S= 141 $\times 10^{-6}$M

30. How many structural isomers of tertiary amines corresponding to molecular formula $C_6H_{15}N$ are possible?

Sol.


Mathematics

1. If a, b and care in G.P. such that x and y are the arithmetic a means between a, b and b, c, respectively, then $\frac{a}{x} + \frac{c}{y}$ is equal to

(a) 0 $\quad$ (b) 1 $\quad$ (c) 2 $\quad$ (d) 1/2

Sol. Given $a,\ b,\ c$ are in GP and x, y are arithmetic mean of a, b and b, c

$b^2 = ac\ldots(1)$

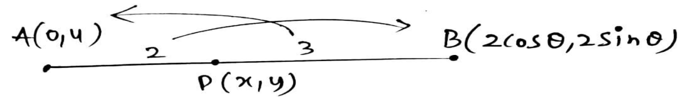
$x = \frac{(a+b)}{2}, y = \frac{(b+c)}{2}$

$$\frac{a}{x} + \frac{c}{y} = \frac{2a}{(a+b)} + \frac{2c}{(b+c)}$$
$$= \frac{[2a(b+c) + 2c(a+b)]}{(a+b)(b+c)}$$
$$= \frac{(2ab+2ac+2ac+2bc)}{(ab+b^2+ac+bc)}$$
$$= 2\frac{(ab+2ac+bc)}{(ab+2ac+bc)} \left(\text{since } b^2 = ac\right)$$
$$= 2$$
$$\Rightarrow \frac{a}{x} + \frac{c}{y} = 2$$

2. Let A = (0, 4) and B = (2 cos θ, 2 sin θ), for some θ. Let P divide the line segment AB in the ratio 2 : 3 internally. The locus of P is

(A) Circle (B) Ellipse (C) Parabola (D) Hyperbola

Sol.



$P = \frac{3A+2B}{3+2}$

$x = \frac{4\cos\theta + 0}{5}$ ....(i)

$y = \frac{4\sin\theta + 12}{5}$ ...(ii)

from (i) $\cos\theta = \frac{5x}{4}$ ....(iii)

from (ii) $\sin\theta = \frac{5y}{4} - 12$ ...(iv)

$(iii)^2 + (iv)^2 \Rightarrow \left(\frac{5x}{4}\right)^2 + \left(\frac{5y}{4} - 12\right)^2 = 1$

$x^2 + \left(y - \frac{48}{5}\right)^2 = \frac{16}{25}$

circle with centre $\left(0, \frac{48}{5}\right)$ and radius $\frac{4}{5}$

3. If A.M. and H.M. of two numbers are 27 and 12, respectively, then G.M. of the two numbers will be

(a) 9 **(b)** 18 (c) 24 (d) 36

4. For a real variable a > 1, consider the points $A_k = (ka, a^k)$, k = 1, 2,..... n in Cartesian plane. If a and ß represent respectively the arithmetic mean of x- coordinates and the geometric mean of y coordinates of A_k then the locus of the point P (α, β) is

(A) $ny = \left(\frac{2x}{n}\right)^{n^2+1}$ **(B)** $y^2 = \left(\frac{2x}{n+1}\right)^{n+1}$ (C) $y = \left(\frac{2x}{n+1}\right)^{n}$ (D) $y = n(n+1)(x-(n+1))$

Sol.

$A_K = (ka, a^k)$

$\alpha = \frac{A_1 + A_2 + \cdots A_n}{n} = \frac{a + 2a + 3a + \cdots na}{n}$

$\alpha = \frac{(n+1)}{2} a$

$\beta = (A_1 \cdot A_2 \cdot A_3 \cdots A_n)^{\frac{1}{n}}$

$= (a \cdot a^2 \cdot a^3 \cdots a^n)^{\frac{1}{n}}$

$= \left(a^{\frac{n(n+1)}{2}}\right)^{\frac{1}{n}} = a^{\frac{n+1}{2}}$

$\beta = \left(\frac{2\alpha}{n+1}\right)^{\frac{n+1}{2}}$

Replace $\alpha \to x$, $\beta \to y$ $\Rightarrow y^2 = \left(\frac{2x}{n+1}\right)^{n+1}$

5. There is a certain sequence of positive real numbers. Beginning from the third term, each term of the se quence is the sum of all the previous terms. The seventh term is equal to 1000 and the first term is equal to 1. The second term of this sequence is equal to

(a) 246 (b) 123/2 (c) 123/4 (d) 124

Sol. **Sequence is $t_1 + t_2 + t_3 + t_4 + \ldots$**

$t_3 = t_1 + t_2,\ t_7 = 1000,\ t_1 = 1$

but $t_7 = t_1 + t_2 + t_3 + t_4 + t_5 + t_6$

$1000 = 2(t_1 + t_2 + t_3 + t_4 + t_5)$

$= 4(t_1 + t_2 + t_3 + t_4)$

$= 8(t_1 + t_2 + t_3)$

$= 16(t_1 + t_2)$

$\therefore\ t_1 + t_2 = 125/2$

$\therefore\ t_2 = 125/2 - 1 = 123/2$

Hence, the correct answer is (b).

6. What is the locus of a point which is equidistant from the point (m + n,n - m) and the point (m - n, n + m)

(A) mx = ny **(B)** nx = - my (C) nx = my (D) mx = -nv

7. The cycloid through the origin, generated by a circle of radius r rolling over the x-axis on the positive side ($y \geq 0$), consists of the points (x, y), with

$$x = r(t - \sin t)$$
$$y = r(1 - \cos t)$$

Then the locus of the cycloid will be

(A) $x = r\cos^{-1}\left(1 - \frac{y}{r}\right) - \sqrt{y(2r-1)}$ (B) $y = r\cos^{-1}\left(1 - \frac{x}{r}\right) - \sqrt{y(2r-x)}$

(C) $x = r(\sin y + \cos y)$ (D) $y = r(\sin x + \cos x)$

Sol. $x = r(t - \sin t)$ —(i)

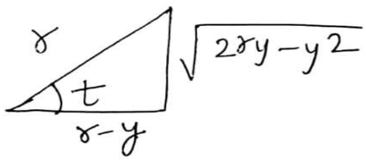
$y = r(1 - \cos t)$ —(ii)

from (ii)

$\cos t = 1 - \frac{y}{r}$

$\Rightarrow t = \cos^{-1}\left(1 - \frac{y}{r}\right)$ .....(iii)

$\sin t = \frac{\sqrt{y(2r-y)}}{r}$



from (i)

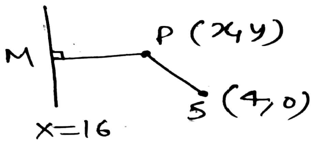
$x = rt - r\sin t$

$\Rightarrow x = r\cos^{-1}\left(1 - \frac{y}{r}\right) - \sqrt{y(2r-y)}$

8. A point moves such that its distance from the point (4, 0) is half that of its distance from the line x = 16 The locus of the point is

(A) $3x^2 + 4y^2 = 192$ (B) $4x^2 + 3y^2 = 192$ (C) $x^2 + y^2 = 192$ (D) None of these

Sol.



$PS = \frac{1}{2} PM$

$\sqrt{(x-4)^2 + y^2} = \frac{1}{2}|x-16|$

$4((x-4)^2 + y^2) = (x-16)^2$

$4x^2 + 4y^2 - 32x + 64 = x^2 - 32x + 256$

$3x^2 + 4y^2 = 192$

9. If $3^{(\log_3 7)^x} = 7^{(\log_7 3)^x}$, then value of x will be

(A) **1/2** (B) 1/4 (C) 1/3 (D) 1

Sol. $3^{(\log_3 7)^x} = 7^{(\log_7 3)^x}$

Take log both sides (base 7)

$\log_7\left(3^{(\log_3 7)^x}\right) = \log_7\left(7^{(\log_7 3)^x}\right)$

$\Rightarrow (\log_3 7)^x \cdot \log_7 3 = (\log_7 3)^x \cdot \log_7 7$

$\Rightarrow (\log_3 7)^x \cdot \frac{1}{\log_3 7} = \frac{1}{(\log_3 7)^x} \cdot 1$

$\Rightarrow (\log_3 7)^{2x} = \log_3 7$

$\Rightarrow x = \frac{1}{2}$

10. If $\cos 6\theta + \cos 4\theta + \cos 2\theta + 1 = 0$, where $0 < \theta < 180^o$, then $\theta =$

(a) $30^o, 45^o$ (b) $45^o, 90^o$ (c) $135^o, 150^o$ (d) $30^o, 45^o, 90^o, 135^o, 150^o$

Sol.

$\cos 6\theta + \cos 4\theta + \cos 2\theta + 1 = 0$

$\Rightarrow 2\cos 4\theta \cdot \cos 2\theta + 2\cos^2 2\theta = 0$

$\Rightarrow 2\cos 2\theta\,(\cos 4\theta + \cos 2\theta) = 0$

$\Rightarrow 2\cos 2\theta \cdot 2\cos 3\theta \cos\theta = 0$

$\Rightarrow$ either $\cos\theta = 0$ or $\cos 2\theta = 0$ or $\cos 3\theta = 0$

$\Rightarrow \theta = \frac{\pi}{2}$, $2\theta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$, $3\theta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ or $\frac{5\pi}{2}$

$\Rightarrow \theta = \frac{\pi}{2}$ $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{6}, \frac{\pi}{2}$ or $\frac{5\pi}{6}$

$\Rightarrow \theta = \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{6}$

11. Solution of $\left|x + \frac{1}{x}\right| < 4$ is

(A) $\left(2-\sqrt{3}, 2+\sqrt{3}\right) \cup \left(-2-\sqrt{3}, -2+\sqrt{3}\right)$ (B) $R - \left(2-\sqrt{3}, 2+\sqrt{3}\right)$

(C) $R - \left(-2-\sqrt{3}, -2+\sqrt{3}\right)$ (D) None of these

Sol.

$\left|x + \frac{1}{x}\right| < 4 \Rightarrow$ $x + \frac{1}{x} < 4$ ... (i) or $x + \frac{1}{x} > -4$ ... (ii)

(i) $x^2 - 4x + 1 < 0$ $\alpha, \beta = \frac{4 \pm \sqrt{12}}{2} = 2 \pm \sqrt{3}$

$\Rightarrow (x - 2 + \sqrt{3})(x - 2 - \sqrt{3}) < 0$

$\Rightarrow x \in (2-\sqrt{3}, 2+\sqrt{3})$

(ii) $x^2 + 4x + 1 > 0$ $\gamma, \delta = -2 \pm \sqrt{3}$

$\Rightarrow (x + 2 - \sqrt{3})(x + 2 + \sqrt{3}) > 0$

$\Rightarrow x \in (-\infty, -2-\sqrt{3}, -2+\sqrt{3}, \infty)$

from (i) & (ii) $x \in R - (-2-\sqrt{3}), (-2+\sqrt{3})$

12. A ray of light coming from the point (1, 2) is reflected at a point A on the x-axis and then passed through the point (5, 3). The coordinates of the point A are

(a) (13/5, 0) (b) (5/13, 0) (c) (-7, 0) (d) None of these

Sol.

13. If $r\sin\theta = 3, r = 4(1+\sin\theta), 0 \le \theta \le 2\pi$ then $\theta =$

(A) $\frac{\pi}{6},\frac{\pi}{3}$ **(B)** $\frac{\pi}{6},\frac{5\pi}{6}$ (C) $\frac{\pi}{3},\frac{\pi}{4}$ (D) $\frac{\pi}{2},\pi$

Sol.

14. Axis of a parabola is $y = x$ and vertex and focus are at a distance $\sqrt{2}$ and $2\sqrt{2}$ respectively from the origin. Then, equation of the parabola is

(a) $(x - y)^2 = 8(x + y - 2)$ (b) $(x + y)^2 = 2(x + y - 2)$

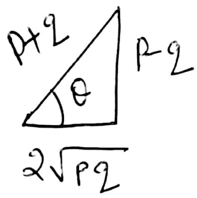
(c) $(x - y)^2 = 4(x + y - 2)$ (d) $(x + y)^2 = 2(x - y + 2)$

15. If $\operatorname{cosec}\theta = \frac{p+q}{p-q}$, then $\cot\left(\frac{\pi}{4}+\frac{\theta}{2}\right) =$

(A) $\sqrt{\frac{p}{q}}$ **(B)** $\sqrt{\frac{q}{p}}$ (C) $\sqrt{pq}$ (D) pq

Sol.
$$\cot\left(\frac{\pi}{4}+\frac{\theta}{2}\right) = \frac{1}{\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)} \quad \cdots (i)$$

$$\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right) = \frac{1+\tan\frac{\theta}{2}}{1-\tan\frac{\theta}{2}} = \frac{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}$$



$$= \frac{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}} \cdot \frac{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}$$

$$= \frac{1+\sin\theta}{\cos\theta} = \sec\theta + \tan\theta$$

$$= \frac{p+q}{2\sqrt{pq}} + \frac{p-q}{2\sqrt{pq}} = \frac{2p}{2\sqrt{pq}} = \sqrt{\frac{p}{q}}$$

16. $\sum_{r=0}^{n} {}^{n}c_r \sin(rx)\cos(n-r)x =$

(A) $2^n \sin(nx)\cos(nx)$ (B) $2^{n-1}\sin(nx)\cos(nx)$ (C) $2^{n-1}\sin(nx)$ (D) $2^{n-1}\cos(nx)$

Sol.

17. If $\cos x = \tan y$, $\cot y = \tan z$ and $\cot z = \tan x$ then $\sin x =$

(A) $\frac{\sqrt{5}+1}{4}$ (B) $\frac{\sqrt{5}-1}{4}$ (C) $\frac{\sqrt{5}+1}{2}$ **(D)** $\frac{\sqrt{5}-1}{2}$

Sol.

18. A rod of fixed length k slides along the coordinate axes. If it meets the axes at A(a,0) and B(0,b) then the minimum value of $\left(a+\frac{1}{a}\right)^2+\left(b+\frac{1}{b}\right)^2$ is

(A) 0 (B) 8 (C) $k^2-4+\frac{4}{k^2}$ (D) $k^2+4+\frac{4}{k^2}$

Sol. $a^2+b^2=k^2$

$\left(a+\frac{1}{a}\right)^2+\left(b+\frac{1}{b}\right)^2$

$=a^2+b^2+4+\frac{1}{a^2}+\frac{1}{b^2}$

$=k^2+4+\frac{k^2}{a^2b^2}$

$\geq k^2+4+\frac{k^2}{\left(\frac{a^2+b^2}{2}\right)^2}$ (As A.M. ≥ G.M.)

$=k^2+4+\frac{4}{k^2}$

19. Locus of middle point of all chords of $\frac{x^2}{4}-\frac{y^2}{9}=1$. Which are at distance of '2' units from vertex of paraboal $y^2=-8ax$ is

(A) $\left(\frac{x^2}{4}+\frac{y^2}{9}\right)=\frac{xy}{6}$ **(B)** $\left(\frac{x^2}{4}-\frac{y^2}{9}\right)^2=4\left(\frac{x^2}{16}+\frac{y^2}{81}\right)$ (C) $\left(\frac{x^2}{4}+\frac{y^2}{9}\right)^2=\left(\frac{x^2}{9}+\frac{y^2}{4}\right)$ (D) None of these

20. Statement 1 : An equation of a common tangent to the parabola $y^2=16\sqrt{3}x$ and the ellipse $2x^2+y^2=4$ and $y=2x+2\sqrt{3}$

Statement 2 : If the line $y=mx+\frac{4\sqrt{3}}{m},(m\neq 0)$ is a common tangent to the parabola $y^2=16\sqrt{3}x$ and the ellipse $2x^2+y^2=4$, then m satisfies $m^4+2m^2=24$

(a) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for Statement 1.
(b) Statement 1 is true, Statement 2 is false.
(c) Statement 1 is false, Statement 2 is true.
(d) Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation for Statement 1.

Sol.

$$y^2 = 16\sqrt{3}x,\ y = mx + \frac{4\sqrt{3}}{m}$$

$$\frac{x^2}{2} + \frac{y^2}{4} = 1,\ x = m_1 y + \sqrt{4m_1^2 + 2}$$

$$\Rightarrow y = \frac{x}{m_1} - \sqrt{4 + \frac{2}{m_1^2}} \quad m = \frac{1}{m_1}$$

Now, $$\left(\frac{4\sqrt{3}}{m}\right)^2 = \left(-\sqrt{4 + \frac{2}{m_1^2}}\right)^2$$

$$\Rightarrow \frac{48}{m^2} = 4 + \frac{2}{m_1^2} = 4 + 2m^2 \Rightarrow \frac{24}{m^2} = 2 + m^2$$

$$\Rightarrow m^4 + 2m^2 - 24 = 0 \quad \ldots(1)$$

$$\Rightarrow (m^2 + 6)(m^2 - 4) = 0 \Rightarrow m = \pm 2$$

Statement 2 : If $y = mx + \frac{4\sqrt{3}}{m}$ is a common tangent to $y^2 = 16\sqrt{3}x$ and ellipse $2x^2 + y^2 = 4$, then m satisfies $m^4 + 2m^2 - 24 = 0$

From (1), statement 2 is a correct explanation for statement 1.

1. The mean square deviations of a set of observations $x_1, x_2, \ldots, x_n$ about a point c is defined to be $\frac{1}{n}\sum_{i=1}^{n}(x_i - c)^2$.

The mean square deviations about -1 and +1 of a set of observations are 7 and 3, respectively. Find the variance of this set of observations.

Sol.

2. The value of $6 + \log_{3/2}\left(\frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\ldots}}}\right)$ is

Sol.

Given, $6 + \log_{\frac{3}{2}}\left(\frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\cdots}}}\right)$

Let $\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{\ldots}}} = y$

$\therefore \quad y = \sqrt{4 - \frac{1}{3\sqrt{2}}y}$

$\Rightarrow \quad y^2 + \frac{1}{3\sqrt{2}}y - 4 = 0$

$\Rightarrow \quad 3\sqrt{2}y^2 + y - 12\sqrt{2} = 0$

$\therefore \quad y = \frac{-1 \pm 17}{6\sqrt{2}}$

or $\quad y = \frac{8}{3\sqrt{2}}$

Now, $6 + \log_{\frac{3}{2}}\left(\frac{1}{3\sqrt{2}} \cdot y\right) = 6 + \log_{\frac{3}{2}}\left(\frac{1}{3\sqrt{2}} \cdot \frac{8}{3\sqrt{2}}\right)$

$= 6 + \log_{\frac{3}{2}}\left(\frac{4}{9}\right) = 6 + \log_{\frac{3}{2}}\left(\frac{3}{2}\right)^{-2}$

$= 6 - 2 \cdot \log_{\frac{3}{2}}\left(\frac{3}{2}\right) = 4$

3. The vertices of a triangle are $\left(pq, \frac{1}{pq}\right), \left(qr, \frac{1}{qr}\right)$ and $\left(\text{rp}, \frac{1}{\text{rp}}\right)$ where p, q and r are the roots of the equation $y^3 - 3y^2 + 6y + 1 = 0$. Find the sum of the coordinates of its centroid.

Sol. p, q and r are the roots of equation $y^3 - 3y^2 + 6y + 1 = 0$.

So, $p + q + r = 3, pq + qr + rp = 6$ and $pqr = -1$

Now, the centroid of the triangle is:

$$\left(\frac{pq+qr+rp}{3}, \frac{\frac{1}{pq}+\frac{1}{qr}+\frac{1}{rp}}{3}\right) \equiv \left(\frac{pq+qr+rp}{3}, \frac{p+q+r}{3pqr}\right) \equiv \left(\frac{6}{3}, \frac{3}{-3}\right) \equiv (2, -1)$$

4. The coefficients of the $(r - 1)^{th}$, r^{th} and $(r + 1)^{th}$ terms in the expansion of $(x + 1)^n$ are in the ratio 1 : 3 : 5. Find n + r _____.

Sol. The coefficients of the $(r - 1)^{th}$, r^{th} and $(r + 1)^{th}$ terms in the expansion of $(x + 1)^n$ are ${}^nC_{r-2}$, ${}^nC_{r-1}$ and ${}^nC_r$ respectively. Since these coefficients are in the ratio 1 : 3 : 5, we have

$$\frac{{}^nC_{r-2}}{{}^nC_{r-1}} = \frac{1}{3} \text{ and } \frac{{}^nC_{r-1}}{{}^nC_r} = \frac{3}{5}$$

$$\frac{{}^nC_{r-2}}{{}^nC_{r-1}} = \frac{r-1}{n-r+2}$$

$$\Rightarrow \quad \frac{r-1}{n-r+2} = \frac{1}{3}$$

$$\Rightarrow \quad n - 4r + 5 = 0$$

$$\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{r}{n-r+1}$$

$$\Rightarrow \quad \frac{r}{n-r+1} = \frac{3}{5}$$

$$\Rightarrow \quad 3n - 8r + 3 = 0$$

Solving (1) and (2), we get

$r = 3$ and $n = 7$

5. The last three digits of the number (6!+1)6!

Sol.

$(720 + 1)^{720}$

$\Rightarrow {}^{720}C_0 720^{720} + \ldots\ldots {}^{720}C_{719}.720.1 + {}^{720}C_{720}.1^{720}$

$\Rightarrow \underbrace{{}^{720}C_0.\, 720^{720} + \ldots\ldots}_{1000K} \underbrace{720.720}_{\ldots.400} + 1$

So we have to check last two terms ... 401 hence last 3 digits are 401

6. $p^4 + q^3 = 2 (p > 0, q > 0)$, if the maximum value of term independent of x in the expansion of $\left(px^{\frac{1}{12}} + qx^{-\frac{1}{9}}\right)^{14}$ is K then find the value of $\frac{2K}{14_{C_5}}$

Sol. $\left(px^{\frac{1}{12}} + qx^{-\frac{1}{9}}\right)^{14}$

General term, $T_{r+1} = {}^{14}C_r \left(px^{\frac{1}{12}}\right)^{14-r} \left(qx^{-\frac{1}{9}}\right)^r = {}^{14}C_r\, p^{14-r} q^r\, x^{\frac{14-r}{12} - \frac{r}{9}}$

For this term to be independent of x, we must have

$$\frac{14-r}{12} - \frac{r}{9} = 0$$

$\therefore \quad r = 6$

$\therefore$ Term independent of $x = {}^{14}C_6\, p^8 q^6 = {}^{14}C_6 (p^4 q^3)^2$

Now p^4 and q^3 are positive.

Using AM $\geq$ GM, we get

$$\frac{p^4 + q^3}{2} \geq (p^4q^3)^{1/2} \Rightarrow (p^4q^3)^2 \leq 1$$

Hence, the maximum value of term independent of x is **${}^{14}C_6$**.

7. If a = $\log_{245}175$ and b = $\log_{1715}875$ then the value of $\frac{1-ab}{a-b}$ is __5___.

Sol.

8. What is the mean of the range, mode and median of the data given below?
5, 10, 3, 6, 4, 8, 9, 3, 15, 2, 9, 4, 19, 11, 4
(A) 10 (B) 12 (C)8 (D) 9

Sol. Arranging the given data in ascending order

2, 3, 3, 4, 4, 4, 5, 6, 8, 9, 9, 10, 11, 15, 19

Here, Most frequent data is 4 so

Mode = 4

Total terms in the given data, (n) = 15 (It is odd)

Median = $\{(n + 1)/2\}^{th}$ term when n is odd

$\Rightarrow \{(15 + 1)/2\}^{th}$ term

$\Rightarrow (8)^{th}$ term

$\Rightarrow$ 6

Now, Range = Maximum value − Minimum value

$\Rightarrow$ 19 − 2 = 17

Mean of Range, Mode and median = (Range + Mode + Median)/3

$\Rightarrow$ (17 + 4 + 6)/3

$\Rightarrow$ 27/3 = 9

∴ The mean of the Range, Mode and Median is 9

9. Let α and ß be the roots of equation $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$, for $n \geq 1$ then the value of

$$\frac{a_{12} - 2a_{10}}{2a_{11}} + \frac{a_{10} - 2a_8}{2a_9} = \_\_\_6\_\_\_\_.$$

Sol. Then $\alpha^2 - 6\alpha - 2 = 0$

Multiplying by α^n it become,

$$\alpha^{n+2} - 6\alpha^{n+1} - 2\alpha^n = 0$$

Similarly, $\beta^{n+2} - 6\beta^{n+1} - 2\beta^n = 0$

Subtracting, we get

$(\alpha^{n+2} - \beta^{n+2}) - 6(\alpha^{n+1} - \beta^{n+1}) - 2(\alpha^n - \beta^n) = 0$

i.e., $a_{n+2} - 6a_{n+1} - 2a_n = 0$

Thus, $\frac{a_{n+2} - 2a_n}{2a_{n+1}} = 3$

$$\frac{a_{12} - 2a_{10}}{2a_{11}} + \frac{a_{10} - 2a_8}{2a_9} = 6$$

10. The foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ coincide. Then the value of b^2 is

Sol. Eccentricity for $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

is $b^2 = a^2(1 - e^2)$

and eccentricity for $\frac{x^2}{\frac{144}{25}} - \frac{y^2}{\frac{81}{25}} = 1$ is

$$e_1 = \frac{a_1^2 + b_1^2}{a_1^2}$$

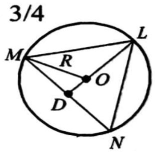
$$\therefore e_1 = \sqrt{1 + \frac{81}{144}} = \frac{15}{12}$$

Again foci $= a_1 e_1 = \frac{12}{5} \times \frac{15}{12} = 3$

$\therefore$ focus of hyperbola is $(3, 0) = (ae, 0)$

So focus of ellipse $(ae, 0) = (4e, 0)$

As their foci are same $\therefore 4e = 3$ $\quad \therefore e = 3/4$



$$\therefore e^2 = 1 - \left(\frac{b}{a}\right)^2 = 1 - \frac{b^2}{16}$$

or $\frac{b^2}{16} = 1 - e^2 = 1 - \frac{9}{16}$

$\Rightarrow b^2 = 7$